Star

NURTURING TODAY'S YOUNG PEOPLE,
INSPIRING TOMORROW'S LEADERS

PRIMARY MATHS CURRICULUM FRAMEWORK

SPRING 2022

Contents

Introduction

1. Mathematics is a key life skill that enables an individual to participate fully as a member of society. Mathematics is a creative and highly inter-connected discipline that has been developed over centuries, providing the solution to some of history's most intriguing problems. It is essential to everyday life, critical to science, technology and engineering, and necessary for financial literacy and most forms of employment. A high-quality mathematics education therefore provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics, and a sense of enjoyment and curiosity about the subject.

2. The aims of our maths curriculum provision are to support pupils to:

    - Have **rich and enjoyable experiences** in mathematics by providing the knowledge, skills and understanding that enable all our pupils to flourish in society and be fully prepared for the next stage in their learning;
    - Develop positive and confident attitudes towards mathematics;
    - **Reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof;
    - **Solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions;
    - Develop the correct **mathematical vocabulary**;
    - Work independently and collaboratively;
    - Use technology to develop mathematical concepts;
    - Use and apply their mathematical knowledge to **real-life contexts**;
    - Become **fluent** in the fundamentals of mathematics, including through **varied** and **frequent** practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.

3. The Star maths curriculum is based on the 2014 expectations and aims of the National Curriculum for mathematics (NC). It incorporates the 'Ready-to-progress' criteria set out in the DfE non-statutory guidance (2020), the 2021 statutory framework for the Early Years Foundation Stage (EYFS) and the Early Years 'Development Matters' (2020) EYFS document.

4. This framework provides a structure for the teaching of mathematics. It sets out the minimum curriculum expectations of each year group, showing how content should be carefully sequenced year on year via the progression map.

5. Consistency across Star schools is essential and thus this framework also sets out in detail the Star commitment to maths mastery.

6. Provision for our pupils who are working at a greater depth within their year group standard is paramount to improving and maintaining high standards, and consequently guidance regarding the development of greater depth mathematicians is also integral to this framework. Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content.

National Curriculum Expectations

7. All Star primary schools ensure that the year group National Curriculum programmes of study and expectations as set out in the EYFS framework are taught and mastered, and that all pupils are given

ample opportunity to work at a greater depth within the standard as set out in those programmes of study.

8. Below are the National Curriculum statutory programmes of study for each year group and the Early Learning Goals (ELGs) for EYFS. The more detailed progression map that follows these is based on these national expectations. The NC and ELGs are ultimately the underpinning source of year group content, and the progression map expands upon this, giving more detail and sequencing content at a strand and sub-strand level. *The use of a scheme is simply to support delivery of our curriculum: it is not the curriculum itself.*

Mathematics guidance: key stages 1 and 2

9. The 2020 DfE document 'Teaching Mathematics in Primary Schools' was produced to support teachers to deliver the National Curriculum more effectively, bringing greater clarity and coherence to it. The guidance identifies the most important conceptual knowledge and understanding that pupils require to progress smoothly from Year 1 to Year 6. These important concepts are referred to as ready-to-progress (RTP) criteria and provide a coherent, linked framework to support pupils' mastery of the primary mathematics curriculum. These are set out below alongside the National Curriculum statements, labelled in blue.

Early Learning Goals for Mathematics

10. As per the EYFS framework, Star Academies believe developing a strong grounding in number is essential so that all children develop the necessary building blocks to excel mathematically. Children should be able to count confidently; develop a deep understanding of the numbers to 10; understand the relationships between numbers to 10; and identify patterns within those numbers. By providing frequent and varied opportunities to build and apply this understanding - such as using manipulatives, including small pebbles and tens frames for organising counting - children will develop a secure base of knowledge and vocabulary from which the mastery of mathematics is built. In addition, it is important that the curriculum includes rich opportunities for children to develop their spatial reasoning skills across all areas of mathematics, including shape, space and measures. It is important that children develop positive attitudes and interests in mathematics, look for patterns and relationships, spot connections, 'have a go', talk to adults and peers about what they notice, whilst not being afraid to make mistakes.

11. Teachers ensure that they reflect the varied ways that young pupils learn in their curriculum delivery. These 'characteristics of effective teaching and learning' are:

    - **Playing and exploring** - children investigate and experience things, and 'have a go';
    - **Active learning** - children concentrate and keep on trying if they encounter difficulties, and enjoy achievements;
    - **Creating and thinking critically** - children have and develop their own ideas, make links between ideas, and develop strategies for doing things.

12. Within the EYFS Framework, the 'specific area' of mathematics has two ELGs:

    - **Number**: Have a deep understanding of number to 10, including the composition of each number. Subitise (recognise quantities without counting) up to 5. Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts.
    - **Numerical Patterns**: Verbally count beyond 20, recognising the pattern of the counting system. Compare quantities up to 10 in different contexts, recognising when one quantity is greater than,

less than or the same as the other quantity. Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.

National Curriculum

Year 1 Programme of Study

Number – number and place value

13. Statutory Requirements (and non-statutory 'Ready-to -progress' criteria)

Pupils should be taught to:

- Count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number;
- Count within 100, forwards and backwards, starting with any number;
- Count, read and write numbers to 100 in numerals;
- Count forwards and backwards in multiples of 2, 5 and 10, up to 10 multiples, beginning with any multiple, and count forwards and backwards through the odd numbers;
- Given a number, identify one more and one less;
- Identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least;
- Read and write numbers from 1 to 20 in numerals and words;
- Reason about the location of numbers to 20 within the linear number system, including comparing using < > and =.

Number – addition and subtraction

14. Statutory Requirements (and non-statutory 'Ready-to -progress' criteria)

Pupils should be taught to:

- Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs;
- Read, write and interpret equations containing addition ( ), subtraction ( ) and equals ( ) symbols, and relate additive expressions and equations to real-life contexts;
- Represent and use number bonds and related subtraction facts within 20;
- Add and subtract one-digit and two-digit numbers to 20, including 0;
- Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = ? – 9;
- Develop fluency in addition and subtraction facts within 10;
- Compose numbers to 10 from 2 parts, and partition numbers to 10 into parts, including recognising odd and even numbers.

Number – multiplication and division

15. Statutory Requirements

Pupils should be taught to:

- Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations, and arrays with the support of the teacher;

Number – Fractions

16. Statutory Requirements

Pupils should be taught to:

- Recognise, find and name a half as one of two equal parts of an object, shape or quantity;
- Recognise, find and name a quarter as one of four equal parts of an object, shape or quantity.

Measurement

17. Statutory Requirements

Pupils should be taught to:

- Compare, describe and solve practical problems for:
  i) Lengths and heights [for example, long/short, longer/shorter, tall/short, double/half];
  ii) Mass/weight [for example, heavy/light, heavier than, lighter than];
  iii) Capacity and volume [for example, full/empty, more than, less than, half, half full, quarter];
  iv) Time [for example, quicker, slower, earlier, later];
- Measure and begin to record the following:
  i. Lengths and heights;
  ii. Mass/weight;
  iii. Capacity and volume;
  iv. Time (hours, minutes, seconds);
- Recognise and know the value of different denominations of coins and notes;
- Sequence events in chronological order using language [for example, before and after, next, first, today, yesterday, tomorrow, morning, afternoon and evening];
- Recognise and use language relating to dates, including days of the week, weeks, months and years;
- Tell the time to the hour and half past the hour and draw the hands on a clock face to show these times.

Geometry – properties of shapes

18. Statutory Requirements (Ready-to-progress criteria)

Pupils should be taught to:

- Recognise and name common 2-D and 3-D shapes, including:
  i. 2-D shapes [for example, rectangles (including squares), circles and triangles];
  ii. 3-D shapes [for example, cuboids (including cubes), pyramids and spheres];
- Recognise common 2D and 3D shapes presented in different orientations, and know that rectangles, triangles, cuboids and pyramids are not always similar to one another;
- Compose 2D and 3D shapes from smaller shapes to match an example, including manipulating shapes to place them in particular orientations.

Geometry – position and direction

19. Statutory Requirements

Pupils should be taught to:

- Describe position, direction and movement, including whole, half, quarter and three quarter-turns.

Year 2 Programme of Study

Number – number and place value

20. Statutory Requirements (and non-statutory 'Ready-to -progress' criteria)

Pupils should be taught to:

- Count in steps of 2, 3, and 5 from 0, and in tens from any number, forward and backward;
- Recognise the place value of each digit in a two-digit number (tens, ones) [and compose and decompose two-digit numbers using standard and non-standard partitioning];
- Identify, represent and estimate numbers using different representations, including the number line;
- Compare and order numbers from 0 up to 100; use <, > and = signs;
- Read and write numbers to at least 100 in numerals and in words;
- Use place value and number facts to solve problems;
- Reason about the location of any two-digit number in the linear number system, including identifying the previous and next multiple of 10.

Number – addition and subtraction

21. Statutory Requirements (and non-statutory 'Ready-to -progress' criteria)

Pupils should be taught to:

- Solve problems with addition and subtraction:
    i. Using concrete objects and pictorial representations, including those involving numbers, quantities and measures;
    ii. Applying their increasing knowledge of mental and written methods;
- Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100;
- Add and subtract numbers using concrete objects, pictorial representations, and mentally, including:
    i. A two-digit number and ones;
    ii. A two-digit number and tens;
    iii. Two two-digit numbers;
    iv. Adding three one-digit numbers;
- Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot;
- Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems;
- Add and subtract across 10;
- Secure fluency in addition and subtraction facts within 10, through continued practice;
- Recognise the subtraction structure of 'difference' and answer questions of the form, "How many more…?";
- Add and subtract within 100 by applying related one-digit addition and subtraction facts: add and subtract only ones or only tens to/from a two-digit number.

Number – multiplication and division

22. Statutory Requirements (and non-statutory 'Ready-to -progress' criteria)

Pupils should be taught to:

- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers;
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs;
- Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot;
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts;
- Recognise repeated addition contexts, representing them with multiplication equations and calculating the product, within the 2, 5 and 10 multiplication tables;
- Relate grouping problems where the number of groups is unknown to multiplication equations with a missing factor, and to division equations (quotitive division).

Number – fractions

23. Statutory Requirements

Pupils should be taught to:

- Recognise, find, name and write fractions 1/3 , 1/4, 2/4 and 3/4 of a length, shape, set of objects or quantity;
- Write simple fractions for example, 1/2 of 6 = 3 and recognise the equivalence of 2/4 and 1/2.

Measurement

24. Statutory Requirements

Pupils should be taught to:

- Choose and use appropriate standard units to estimate and measure length/height in any direction (m/cm); mass (kg/g); temperature (°C); capacity (litres/ml) to the nearest appropriate unit, using rulers, scales, thermometers and measuring vessels;
- Compare and order lengths, mass, volume/capacity and record the results using >, < and =;
- Recognise and use symbols for pounds (£) and pence (p); combine amounts to make a particular value;
- Find different combinations of coins that equal the same amounts of money;
- Solve simple problems in a practical context involving addition and subtraction of money of the same unit, including giving change;
- Compare and sequence intervals of time;
- Tell and write the time to five minutes, including quarter past/to the hour and draw the hands on a clock face to show these times;
- Know the number of minutes in an hour and the number of hours in a day.

Geometry – properties of shapes

25. Statutory Requirements (and non-statutory 'Ready-to -progress' criteria)

Pupils should be taught to:

- Identify and describe the properties of 2-D shapes, including the number of sides and line symmetry in a vertical line;

- Identify and describe the properties of 3-D shapes, including the number of edges, vertices and faces;
- Identify 2-D shapes on the surface of 3-D shapes, [for example, a circle on a cylinder and a triangle on a pyramid];
- Compare and sort common 2-D and 3-D shapes and everyday objects;
- Use precise language to describe the properties of 2D and 3D shapes and compare shapes by reasoning about similarities and differences in properties.

Geometry – position and direction

26. Statutory Requirements

Pupils should be taught to:

- Order and arrange combinations of mathematical objects in patterns and sequences;
- Use mathematical vocabulary to describe position, direction and movement, including movement in a straight line and distinguishing between rotation as a turn and in terms of right angles for quarter, half and three-quarter turns (clockwise and anticlockwise).

Statistics

27. Statutory Requirements

Pupils should be taught to:

- Interpret and construct simple pictograms, tally charts, block diagrams and simple tables;
- Ask and answer simple questions by counting the number of objects in each category and sorting the categories by quantity;
- Ask and answer questions about totalling and comparing categorical data.

Year 3 Programme of Study

Number – number and place value

28. Statutory Requirements (and non-statutory 'Ready-to -progress' criteria)

Pupils should be taught to:

- Count from 0 in multiples of 4, 8, 50 and 100; find 10 or 100 more or less than a given number;
- Recognise the place value of each digit in a three-digit number (hundreds, tens, ones) [and compose and decompose three-digit numbers using standard and non-standard partitioning];
- Compare and order numbers up to 1000;
- Identify, represent and estimate numbers using different representation;
- Reason about the location of any three-digit number in the linear number system, including identifying the previous and next multiple of 100 and 10;
- Read and write numbers up to 1000 in numerals and in words;
- Know that 10 tens are equivalent to 1 hundred, and that 100 is 10 times the size of 10; apply this to identify and work out how many 10s there are in other three-digit multiples of 10;
- Divide 100 into 2, 4, 5 and 10 equal parts, and read scales/number lines marked in multiples of 100 with 2, 4, 5 and 10 equal parts;
- Solve number problems and practical problems involving these ideas.

Number – addition and subtraction

29. Statutory Requirements (and non-statutory 'Ready-to -progress' criteria)

Pupils should be taught to:

- Add and subtract numbers mentally, including:
  i. A three-digit number and ones;
  ii. A three-digit number and tens;
  iii. A three-digit number and hundreds;
- Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction;
- Estimate the answer to a calculation and use inverse operations to check answers;
- Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction;
- Secure fluency in addition and subtraction facts that bridge 10, through continued practice;
- Apply place-value knowledge to known additive and multiplicative number facts (scaling facts by 10);
- Calculate complements to 100;
- Manipulate the additive relationship: understand the inverse relationship between addition and subtraction, and how both relate to the part–part–whole structure. Understand and use the commutative property of addition, and understand the related property for subtraction.

Number – multiplication and division

30. Statutory Requirements (and non-statutory 'Ready-to -progress' criteria)

Pupils should be taught to:

- Recall multiplication facts, and corresponding division facts, in the 10, 5, 2, 4, 8 and 3 multiplication tables, and recognise products in these multiplication tables as multiples of the corresponding number;
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods;
- Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects;
- Apply place-value knowledge to known additive and multiplicative number facts (scaling facts by 10);
- Apply known multiplication and division facts to solve contextual problems with different structures, including quotitive and partitive division.

Number – fractions

31. Statutory Requirements (and non-statutory 'Ready-to -progress' criteria)

Pupils should be taught to:

- Count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10; Recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators [find unit fractions of quantities using known division facts (multiplication tables fluency)];

- Recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators;
- Interpret and write proper fractions to represent 1 or several parts of a whole that is divided into equal parts;
- Recognise and show, using diagrams, equivalent fractions with small denominators;
- Add and subtract fractions with the same denominator within one whole [for example, 5/7 + 1/7 = 6/7];
- Compare and order unit fractions, and fractions with the same denominators;
- Reason about the location of any fraction within 1 in the linear number system;
- Solve problems that involve all of the above.

Measurement

32. Statutory Requirements

Pupils should be taught to:

- Measure, compare, add and subtract: lengths (m/cm/mm); mass (kg/g); volume/capacity (l/ml);
- Measure the perimeter of simple 2-D shapes;
- Add and subtract amounts of money to give change, using both £ and p in practical contexts;
- Tell and write the time from an analogue clock, including using Roman numerals from I to XII, and 12-hour and 24-hour clocks;
- Estimate and read time with increasing accuracy to the nearest minute; record and compare time in terms of seconds, minutes and hours; use vocabulary such as o'clock, a.m./p.m., morning, afternoon, noon and midnight;
- Know the number of seconds in a minute and the number of days in each month, year and leap year;
- Compare durations of events [for example to calculate the time taken by particular events or tasks].

Geometry – properties of shapes

33. Statutory Requirements (and non-statutory 'Ready-to -progress' criteria)

Pupils should be taught to:

- Draw 2-D shapes and make 3-D shapes using modelling materials; recognise 3-D shapes in different orientations and describe them;
- Recognise angles as a property of shape or a description of a turn;
- Identify right angles, recognise that two right angles make a half-turn, three make three quarters of a turn and four a complete turn; identify whether angles are greater than or less than a right angle [and identify right angles in 2D shapes presented in different orientations];
- Identify horizontal and vertical lines and pairs of perpendicular and parallel lines;
- Draw polygons by joining marked points and identify parallel and perpendicular sides.

Statistics

34. Statutory Requirements

Pupils should be taught to:

- Interpret and present data using bar charts, pictograms and tables;

- Solve one-step and two-step questions [for example, 'How many more?' and 'How many fewer?'] using information presented in scaled bar charts and pictograms and tables.

Year 4 Programme of Study

Number – number and place value

35. Statutory Requirements (and non-statutory 'Ready-to -progress' criteria)

Pupils should be taught to:

- Count in multiples of 6, 7, 9, 25 and 1000;
- Find 1000 more or less than a given number;
- Count backwards through zero to include negative numbers;
- Recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones) [and compose and decompose four-digit numbers using standard and non-standard partitioning];
- Order and compare numbers beyond 1000;
- Identify, represent and estimate numbers using different representations;
- Round any number to the nearest 10, 100 or 1000;
- Reason about the location of any four-digit number in the linear number system, including identifying the previous and next multiple of 1,000 and 100, and rounding to the nearest of each;
- Solve number and practical problems that involve all of the above and with increasingly large positive numbers;
- Read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value;
- Know that 10 hundreds are equivalent to 1 thousand, and that 1,000 is 10 times the size of 100; apply this to identify and work out how many 100s there are in other four-digit multiples of 100;
- Divide 1,000 into 2, 4, 5 and 10 equal parts, and read scales/number lines marked in multiples of 1,000 with 2, 4, 5 and 10 equal parts.

Number – addition and subtraction

36. Statutory Requirements (and non-statutory 'Ready-to -progress' criteria)

Pupils should be taught to:

- Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate;
- Estimate and use inverse operations to check answers to a calculation;
- Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why;
- Apply place-value knowledge to known additive and multiplicative number facts (scaling facts by 100).

Number – multiplication and division

37. Statutory Requirements (and non-statutory 'Ready-to -progress' criteria)

Pupils should be taught to:

- Recall multiplication and division facts for multiplication tables up to 12 × 12 [and recognise products in multiplication tables as multiples of the corresponding number];

- Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers;
- Recognise and use factor pairs and commutativity in mental calculations;
- Multiply two-digit and three-digit numbers by a one-digit number using formal written layout;
- Solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects;
- Solve division problems, with two-digit dividends and one-digit divisors, that involve remainders, and interpret remainders appropriately according to the context;
- Apply place-value knowledge to known additive and multiplicative number facts (scaling facts by 100);
- Multiply and divide whole numbers by 10 and 100 (keeping to whole number quotients); understand this as equivalent to making a number 10 or 100 times the size.

Number – fractions (including decimals)

38. Statutory Requirements (and non-statutory 'Ready-to -progress' criteria)

Pupils should be taught to:

- Recognise and show, using diagrams, families of common equivalent fractions;
- Count up and down in hundredths; recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten;
- Solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number;
- Add and subtract fractions with the same denominator;
- Add and subtract improper and mixed fractions with the same denominator, including bridging whole numbers;
- Recognise and write decimal equivalents of any number of tenths or hundredths;
- Recognise and write decimal equivalents to 1/4 , 1/2 and 3/4;
- Find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths;
- Round decimals with one decimal place to the nearest whole number;
- Compare numbers with the same number of decimal places up to two decimal places;
- Solve simple measure and money problems involving fractions and decimals to two decimal places;
- Reason about the location of mixed numbers in the linear number system;
- Convert mixed numbers to improper fractions and vice versa.

Measurement

39. Statutory Requirements

Pupils should be taught to:

- Convert between different units of measure [for example, kilometre to metre; hour to minute];
- Measure and calculate the perimeter of a rectilinear figure (including squares) in centimetres and metres;
- Find the area of rectilinear shapes by counting squares;

- Estimate, compare and calculate different measures, including money in pounds and pence.

Geometry – properties of shapes

40. Statutory Requirements (and non-statutory 'Ready-to -progress' criteria)

Pupils should be taught to:

- Compare and classify geometric shapes, including quadrilaterals and triangles, based on their properties and sizes;
- Identify acute and obtuse angles and compare and order angles up to two right angles by size;
- Identify lines of symmetry in 2-D shapes presented in different orientations;
- Complete a simple symmetric figure with respect to a specific line of symmetry;
- Identify regular polygons, including equilateral triangles and squares, as those in which the side-lengths are equal and the angles are equal.

Geometry – position and direction

41. Statutory Requirements

Pupils should be taught to:

- Describe positions on a 2-D grid as coordinates in the first quadrant;
- Describe movements between positions as translations of a given unit to the left/right and up/down;
- Draw polygons, specified by coordinates in the first quadrant, and translate within the first quadrant;
- Reflect shapes in a line of symmetry.

Statistics

42. Statutory Requirements

Pupils should be taught to:

- Interpret and present discrete and continuous data using appropriate graphical methods, including bar charts and time graphs;

  Solve comparison, sum and difference problems using information presented in bar charts, pictograms, tables and other graphs.

Year 5 Programme of Study

Number – number and place value

43. Statutory Requirements

Pupils should be taught to:

- Read, write, order and compare numbers to at least 1,000,000 and determine the value of each digit;
- Count forwards or backwards in steps of powers of 10 for any given number up to 1,000,000;
- Interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers, including through zero;
- Round any number up to 1,000,000 to the nearest 10, 100, 1000, 10,000 and 100,000;
- Solve number problems and practical problems that involve all of the above;
- Read Roman numerals to 1000 (M) and recognise years written in Roman numerals.

Number – addition and subtraction

44. Statutory Requirements (and non-statutory 'Ready-to -progress' criteria)

Pupils should be taught to:

- Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction);
- Add and subtract numbers mentally with increasingly large numbers;
- Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy;
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why;
- Apply place-value knowledge to known additive and multiplicative number facts (scaling facts by 1 tenth or 1 hundredth).

Number – multiplication and division

45. Statutory Requirements (and non-statutory 'Ready-to -progress' criteria)

Pupils should be taught to:

- Identify multiples and factors, including finding all factor pairs of a number [including common factors and common multiples, and express a given number as a product of 2 or 3 factors];
- Know and use the vocabulary of prime numbers, prime factors and composite (nonprime) numbers;
- Establish whether a number up to 100 is prime and recall prime numbers up to 19;
- Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers;
- Multiply and divide numbers mentally drawing upon known facts;
- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context;
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000;
- Recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3});
- Solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes;
- Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign;
- Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates;
- Secure fluency in multiplication table facts, and corresponding division facts, through continued practice;
- Apply place-value knowledge to known additive and multiplicative number facts (scaling facts by 1 tenth or 1 hundredth).

Number – fractions (including decimals and percentages)

46. Statutory Requirements (and non-statutory 'Ready-to -progress' criteria)

Pupils should be taught to:

- Find non-unit fractions of quantities;
- Compare and order fractions whose denominators are all multiples of the same number;
- Identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths [and understand that they have the same value and the same position in the linear number system];
- Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number [for example, 2/5 + 4/5 = 6/5 = 1 1/5];
- Add and subtract fractions with the same denominator and denominators that are multiples of the same number;
- Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams;
- Read and write decimal numbers as fractions [for example, 0.71 = 71/100] [Recall decimal fraction equivalents for ½ , ¼ , 1/5 and 1/10, and for multiples of these proper fractions];
- Recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents. Know that:
  I. 10 tenths are equivalent to 1 one, and that 1 is 10 times the size of 0.1;
  II. 100 hundredths are equivalent to 1 one, and that 1 is 100 times the size of 0.01;
  III. 10 hundredths are equivalent to 1 tenth, and that 0.1 is 10 times the size of 0.01;
- Round decimals with two decimal places to the nearest whole number and to one decimal place;
- Reason about the location of any number with up to 2 decimals places in the linear number system, including identifying the previous and next multiple of 1 and 0.1 and rounding to the nearest of each;
- Read, write, order and compare numbers with up to three decimal places;
- Recognise the place value of each digit in numbers with up to 2 decimal places, and compose and decompose numbers with up to 2 decimal places using standard and non-standard partitioning;
- Solve problems involving number up to three decimal places;
- Recognise the per cent symbol (%) and understand that per cent relates to 'number of parts per hundred', and write percentages as a fraction with denominator 100, and as a decimal;
- Solve problems which require knowing percentage and decimal equivalents of 1/2, 1/4, 1/5, 2/5, 4/5 and those fractions with a denominator of a multiple of 10 or 25;
- Divide 1 into 2, 4, 5 and 10 equal parts, and read scales/number lines marked in units of 1 with 2, 4, 5 and 10 equal parts.

Measurement

47. Statutory Requirements (and non-statutory 'Ready-to -progress' criteria)

Pupils should be taught to:

- Convert between different units of metric measure (for example, kilometre and metre; centimetre and metre; centimetre and millimetre; gram and kilogram; litre and millilitre) [including using common decimals and fractions];
- Understand and use approximate equivalences between metric units and common imperial units such as inches, pounds and pints;
- Measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres;
- Calculate and compare the area of rectangles (including squares), and including using standard units, square centimetres (cm^2) and square metres (m^2) and estimate the area of irregular shapes;

- Estimate volume [for example, using 1 cm^3 blocks to build cuboids (including cubes)] and capacity [for example, using water];
- Solve problems involving converting between units of time;
- Use all four operations to solve problems involving measure [for example, length, mass, volume, money] using decimal notation, including scaling.

Geometry – properties of shapes

48. Statutory Requirements (and non-statutory 'Ready-to -progress' criteria)

Pupils should be taught to:

- Identify 3-D shapes, including cubes and other cuboids, from 2-D representations;
- Know angles are measured in degrees: estimate and compare acute, obtuse and reflex angles;
- Draw given angles, and measure them in degrees (°);
- Identify:
  i. Angles at a point and one whole turn (total 360°);
  ii. Angles at a point on a straight line and 1/2 a turn (total 180°);
  iii. Other multiples of 90°;
- Use the properties of rectangles to deduce related facts and find missing lengths and angles;
- Distinguish between regular and irregular polygons based on reasoning about equal sides and angles.

Geometry – position and direction

49. Statutory Requirements

Pupils should be taught to:

- Identify, describe and represent the position of a shape following a reflection or translation, using the appropriate language, and know that the shape has not changed.

Statistics

50. Statutory Requirements

Pupils should be taught to:

- Solve comparison, sum and difference problems using information presented in a line graph;
- Complete, read and interpret information in tables, including timetables.

Year 6 Programme of Study

Number – number and place value

51. Statutory Requirements (and non-statutory 'Ready-to -progress' criteria)

Pupils should be taught to:

- Read, write, order and compare numbers up to 10,000,000 and determine the value of each digit [*and* compose and decompose numbers up to 10 million using standard and non-standard partitioning];
- Reason about the location of any number up to 10 million, including decimal fractions, in the linear number system, and round numbers (to a required degree of accuracy), as appropriate, including in contexts;

- Use negative numbers in context, and calculate intervals across zero;
- Solve number and practical problems that involve all of the above;
- Divide powers of 10, from 1 hundredth to 10 million, into 2, 4, 5 and 10 equal parts, and read scales/number lines with labelled intervals divided into 2, 4, 5 and 10 equal parts.

Number – addition, subtraction, multiplication and division

52. Statutory Requirements (and non-statutory 'Ready-to -progress' criteria)

Pupils should be taught to:

- Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication;
- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context;
- Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context;
- Perform mental calculations, including with mixed operations and large numbers;
- Identify common factors, common multiples and prime numbers;
- Use their knowledge of the order of operations to carry out calculations involving the four operations;
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why;
- Solve problems involving addition, subtraction, multiplication and division;
- Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy;
- Understand that 2 numbers can be related additively or multiplicatively, and quantify additive and multiplicative relationships (multiplicative relationships restricted to multiplication by a whole number);
- Use a given additive or multiplicative calculation to derive or complete a related calculation, using arithmetic properties, inverse relationships, and place-value understanding;

Number – fractions (including decimals and percentages)

53. Statutory Requirements (and non-statutory 'Ready-to -progress' criteria)

Pupils should be taught to:

- Use common factors to simplify fractions; use common multiples to express fractions in the same denomination;
- Express fractions in a common denomination and use this to compare and order fractions that are similar in value;
- Compare and order fractions with different denominators, including fractions greater than 1, using reasoning, and choose between reasoning and common denomination as a comparison strategy;
- Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions;
- Multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, 1/4 × 1/2 = 1/8];
- Divide proper fractions by whole numbers [for example, 1/3 ÷ 2 = 1/6];

- Associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example, 3/8];
- Identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places;
- Multiply one-digit numbers with up to 2 decimal places by whole numbers;
- Use written division methods in cases where the answer has up to 2 decimal places;
- Solve problems which require answers to be rounded to specified degrees of accuracy;
- Recall and use equivalences between simple fractions, decimals and percentages, including in different contexts.

Number – ratio and proportion

54. Statutory Requirements (and non-statutory 'Ready-to -progress' criteria)

Pupils should be taught to:

- Solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts;
- Solve problems involving the calculation of percentages [for example, of measures, and such as 15% of 360] and the use of percentages for comparison;
- Solve problems involving similar shapes where the scale factor is known or can be found;
- Solve problems involving unequal sharing and grouping using knowledge of fractions and multiples;
- Solve problems involving ratio relationships.

Algebra

55. Statutory Requirements (and non-statutory 'Ready-to -progress' criteria)

Pupils should be taught to:

- Use simple formulae;
- Generate and describe linear number sequences;
- Express missing number problems algebraically;
- Find pairs of numbers that satisfy an equation with two unknowns;
- Enumerate possibilities of combinations of two variables.

Measurement

56. Statutory Requirements

Pupils should be taught to:

- Solve problems involving the calculation and conversion of units of measure, using decimal notation up to three decimal places where appropriate;
- Use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to up to three decimal places;
- Convert between miles and kilometres;
- Recognise that shapes with the same areas can have different perimeters and vice versa;
- Recognise when it is possible to use formulae for area and volume of shapes;
- Calculate the area of parallelograms and triangles;

- Calculate, estimate and compare volume of cubes and cuboids using standard units, including cubic centimetres (cm^3) and cubic metres (m^3), and extending to other units [for example, mm^3 and km^3].

Geometry – properties of shapes

57. Statutory Requirements (and non-statutory 'Ready-to -progress' criteria)

    Pupils should be taught to:

- Draw, compose, and decompose shapes according to given properties, including dimensions, angles and area, and solve related problems;
- Recognise, describe and build simple 3-D shapes, including making nets;
- Compare and classify geometric shapes based on their properties and sizes and find unknown angles in any triangles, quadrilaterals, and regular polygons;
- Illustrate and name parts of circles, including radius, diameter and circumference and know that the diameter is twice the radius;
- Recognise angles where they meet at a point, are on a straight line, or are vertically opposite, and find missing angles.

Geometry – position and direction

58. Statutory Requirements

    Pupils should be taught to:

- Describe positions on the full coordinate grid (all four quadrants);
- Draw and translate simple shapes on the coordinate plane, and reflect them in the axes.

Statistics

59. Statutory Requirements

    Pupils should be taught to:

- Interpret and construct pie charts and line graphs and use these to solve problems;
- Calculate and interpret the mean as an average.

Types of Mathematics Knowledge

60. The Government Research Review Series: Mathematics publication classifies mathematical content into three different types of knowledge for the purposes of explaining how pupils' mathematics schema should be developed over time. These three types of knowledge are:

61. **Declarative knowledge** is static in nature and consists of facts, formulae, concepts, principles and rules. All content in this category can be prefaced with the sentence stem 'I know that' - e.g. I know that 8 x 5 = 40; I know that squares have four right angles.

62. **Procedural knowledge** is recalled as a sequence of steps. The category includes methods and procedures, such as how to perform long division or add fractions with the same denominator. All content in this category can be prefaced by the sentence stem 'I know how' - e.g. I know how to do column addition; I know how to calculate the perimeter of a square.

63. **Conditional knowledge** gives pupils the ability to reason and solve problems. Useful combinations of declarative and procedural knowledge are transformed into strategies when pupils learn to match the problem types that they can be used for. All content in this category can be prefaced by the sentence stem 'I know when' - e.g. I know when to use a bar model to solve a one-step money problem.

64. Over time, as pupils learn and use declarative, procedural and conditional knowledge, the knowledge of relationships between concepts develops and their mathematics schema thus deepen.

65. All Star primary mathematics lessons should be driven by success criteria that are also categorised by these three types of knowledge. These success criteria should not be overly specific, so they apply to the whole lesson, where possible. For example:


Problem solving – multiplication

Discover

1 a) How many people are going to Bolton Towers?
b) How many people are going on a trip today?

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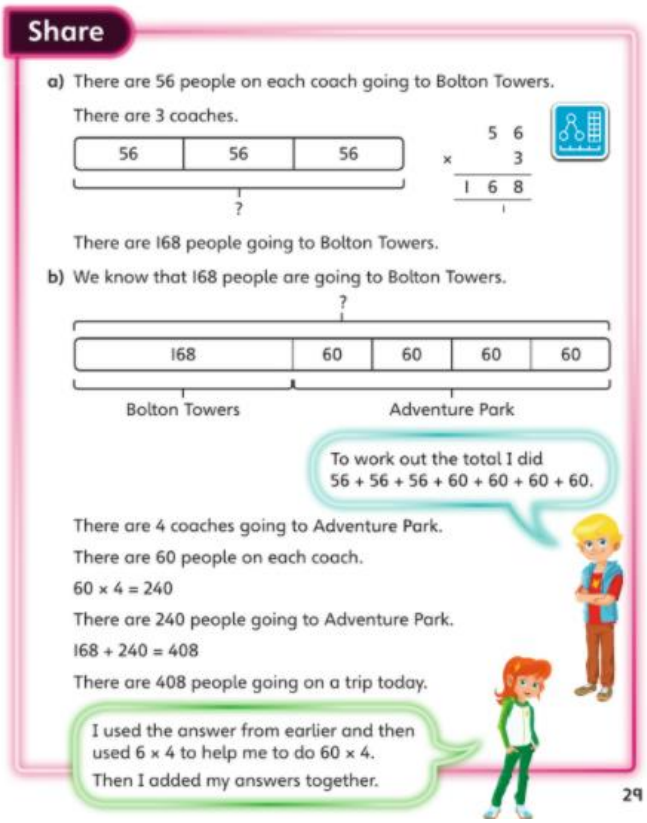
Share

a) There are 56 people on each coach going to Bolton Towers.
There are 3 coaches.

$$\begin{array}{r} 56 \\ \times\ 3 \\ \hline 168 \end{array}$$

There are 168 people going to Bolton Towers.

b) We know that 168 people are going to Bolton Towers.

To work out the total I did 56 + 56 + 56 + 60 + 60 + 60 + 60.

There are 4 coaches going to Adventure Park.
There are 60 people on each coach.
60 × 4 = 240
There are 240 people going to Adventure Park.
168 + 240 = 408
There are 408 people going on a trip today.

I used the answer from earlier and then used 6 × 4 to help me to do 60 × 4. Then I added my answers together.

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66. The success criteria for this type of problem would be:

- I know my times table facts *(declarative knowledge).*
- I know how to do short multiplication *(procedural knowledge).*
- I know how to do column addition *(procedural knowledge).*
- I know when to use a bar model to solve two-step word problems *(conditional knowledge).*

Curriculum Sequencing

67. Sequencing the learning of knowledge within a unit of learning requires careful consideration. The required facts - *declarative knowledge* (e.g. times table facts) - need to be learnt and easily recalled, before using these when learning and mastering the *procedural knowledge* (e.g. short multiplication).

68. Once pupils are proficient in the declarative and procedural knowledge, pupils should then move onto tackling the problem solving and reasoning strategies - *the conditional knowledge*. In short, proficiency in conditional knowledge requires proficiency in associated procedural knowledge, which in itself requires fluency in the underlying declarative knowledge. Problem-solving (conditional knowledge) requires pupils to hold a line of thought. It is not easy to learn, rehearse or experience if the facts and methods that form part of a strategy for solving a problem type are unfamiliar and take up too much working memory.

Declarative Knowledge → **Procedural Knowledge** → **Conditional Knowledge**

(e.g. times table facts) *(e.g. short multiplication)* *(e.g. word problem strategies)*

69. Here, the regular, systematic rehearsal of core declarative knowledge and procedural knowledge, ensured by routinely revisiting this knowledge - e.g. times table practice and discrete age-related arithmetic sessions -, in turn allows pupils to experience success when problem solving and reasoning. Pupils need procedural fluency to enable them to become proficient problem solvers. See chapter entitled *'Regular Practice of Core Skills'* for details.

70. For example, if a learning sequence in Year 4 multiplication culminates in the development of problem solving strategies involving short multiplication and the 3, 4 and 8 times tables, then during those few weeks, pupils should be given regular discrete opportunities to recall the 3, 4 and 8 times table facts, as well as to practice the procedure of short multiplication, using those facts. *Do it Now activities, times table recall sessions and discrete arithmetic foci should all therefore aim to support fluency in underlying age-related declarative and procedural knowledge.*

71. Proficient problem-solving is highly organised, drawing on a well-connected knowledge base of facts, methods and strategies that have been used to solve problems with a similar deep structure before. Successful problem-solving is therefore not just an activity but an *outcome of successful learning of the facts and methods*, and their useful combinations as strategies. Conversely, if a problem-solver does not have conditional knowledge, they are more likely to be distracted by the surface features of problems. *Therefore, pupils need to be fluent with the relevant facts and methods before being expected to learn how to apply them to problem-solving conditions.*

72. Solving word problems should be explicitly modelled and taught to the whole class, where the deep structures of particular types of world problem are converted into modelled strategies (conditional knowledge). Problem solving and reasoning should not simply be given to those pupils who gain new procedural knowledge quickly by way of extending learning - leading to a situation where fast finishers and the more able gain experience in certain types of word problems, whereas others are simply fed a diet over-rich on procedural fluency. *Whole lessons are dedicated to modelling and practicing strategies to tackle particular types of problem - problem solving strategies are an entitlement for all, rather than an 'add-on'.*

Progression Map

	Reception Year	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	Number and Place Value						
Counting	Children count reliably with numbers from one to 20 and place them in order Understand the 'one more than/one less than' relationship between consecutive numbers Verbally count beyond 20, recognising the pattern of the counting system	Count within 100, forwards and backwards, starting with any number Count to and across 100, forward and backwards, beginning with 0 or 1, or from any given number [including ordering – e.g. first, second, third] Given a number 1-100, identify one more/less Count forwards and backwards in multiples of 2, 5 and 10, up to 10 multiples, beginning with any multiple, and count forwards and backwards through the odd numbers	Count in steps of 2, 3 and 5, from 0, and in tens from any number, forward or backward Find 10 more or less than a given number	Count from 0 in multiples of 4, 8, 50 and 100 Find 10 or 100 more or less than a given number	Count in multiples of 6, 7, 9, 25 and 1,000; count in steps of 10 and 100 form any given number beyond 1000 Find 1,000 more or less than a given number	Count forwards or backwards in steps of powers of 10 for any given number up to 1,000,000	
Reading and Writing Numbers	Read and write numbers to 20 in numerals	Read and write numbers to 100 in numerals; read and write numbers from 1 to 20 in numerals and words	Read and write numbers to at least 100 in numerals and in words	Read and write numbers to at least 1000 in numerals and in words	Read and write numbers to at least 10,000 in numerals and in words Read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value	Read and write numbers to at least 1,000,000 in numerals and words Read Roman numerals to 1,000 (M) and recognise years written in Roman numerals	Read and write numbers up to 10,000,000 in numerals and words

	Reception Year	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	Number and Place Value						
Identifying, Estimating, and Representing Numbers	Have a deep understanding of number to 10, including the composition of each number. Subitise (recognise quantities without counting) up to 5. Identify, represent and estimate numbers 1-20	Identify and represent numbers up to 100 using objects and pictorial representations including the number line. Reason about the location of numbers to 20 within the linear number system.	Identify, represent and estimate numbers up to 100 using different representations, including the number line (e.g. place value cards; place value counters; dienes apparatus; number lines) Reason about the location of any two-digit number in the linear number system, including identifying the previous and next multiple of 10.	Identify, represent and estimate numbers up to 1000 using different representations (e.g. place value cards; place value counters; dienes apparatus; number lines) Reason about the location of any three-digit number in the linear number system, including identifying the previous and next multiple of 100 and 10 Divide 100 into 2, 4, 5 and 10 equal parts, and read scales/number lines marked in multiples of 100 with 2, 4, 5 and 10 equal parts.	Identify, represent and estimate numbers up to 10,000 using different representations (e.g. place value cards; place value counters; dienes apparatus; number lines) Reason about the location of any four-digit number in the linear number system, including identifying the previous and next multiple of 1,000 and 100. Divide 1,000 into 2, 4, 5 and 10 equal parts, and read scales/number lines marked in multiples of 1,000 with 2, 4, 5 and 10 equal parts.	Reason about the location of any number with up to 2 decimals places in the linear number system, including identifying the previous and next multiple of 1 and 0.1 Divide 1 into 2, 4, 5 and 10 equal parts, and read scales/number lines marked in units of 1 with 2, 4, 5 and 10 equal parts.	Reason about the location of any number up to 10 million, including decimal fractions, in the linear number system, and round numbers, as appropriate, including in contexts. Divide powers of 10, from 1 hundredth to 10 million, into 2, 4, 5 and 10 equal parts, and read scales/number lines with labelled intervals divided into 2, 4, 5 and 10 equal parts.
Comparing Numbers	Compare quantities up to 10 in different contexts, recognising when one quantity is greater than, less than or the same as the other quantity. Use the language of 'more' and 'fewer' to compare two sets of objects	Use the language of: equal to, more than, less than (fewer), most, least including comparing using < > and =	Compare and order numbers from 0 up to 100; use <, > and = signs	Compare and order numbers up to 1,000 (including using the < and > signs)	Order and compare numbers beyond 1,000 (including using the < and > signs)	Order and compare numbers to at least 1,000,000 (including using the < and > signs)	Order and compare numbers up to 10,000,000 (including using the < and > signs)

	Reception Year	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	Number and Place Value						
Place Value		Begin to recognise the place value in numbers beyond 20 by reading, writing and comparing numbers, supported by concrete and pictorial representations	Recognise the place value of each digit in a two-digit number (tens and ones), including partitioning numbers in different ways (e.g. 23 = 20 + 3 and 23 = 10 + 13) and beginning to understand 0 as a place holder.	Recognise the place value of each digit in a three-digit number (hundreds, tens, ones), including partitioning numbers in different ways (e.g. 146 = 100 + 40 and 6, 146 = 130 + 16) Know that 10 tens are equivalent to 1 hundred, and that 100 is 10 times the size of 10; apply this to identify and work out how many 10s there are in other three-digit multiples of 10.	Recognise the place value of each digit in a four-digit number (e.g. say that the '4' in 3467 is worth '400') Compose and decompose four-digit numbers using standard and non-standard partitioning. Know that 10 hundreds are equivalent to 1 thousand, and that 1,000 is 10 times the size of 100; apply this to identify and work out how many 100s there are in other four-digit multiples of 100.	Determine the value of each digit in numbers up to 1,000,000. Compose and decompose numbers up to 1,00,000 using standard and non-standard partitioning. .	Recognise the place value of each digit in numbers up to 10 million, including decimal fractions, and compose and decompose numbers up to 10 million using standard and non-standard partitioning.

	Reception Year	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	Number and Place Value						
Rounding Numbers					Reason about the location of any four-digit number in the linear number system, including identifying the previous and next multiple of 1,000, 100 and 10 and rounding to the nearest of each.	Round any number up to 1,000,000 to the nearest 10, 100, 1,000, 10,000 and 100,000 Reason about the location of any number with up to 2 decimals places in the linear number system, including identifying the previous and next multiple of 1 and 0.1 and rounding to the nearest of each.	Round any whole number to a required Reason about the location of any number up to 10 million, including decimal fractions, in the linear number system, and round numbers, as appropriate, including in contexts.
Negative Numbers					Count backwards through zero to include negative numbers	Interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers, including through zero	Use negative numbers in context, and calculate intervals across zero
Problem Solving	Solve problems involving all of the above	Solve problems involving all of the above	Use place value and number facts to solve problems involving all of the above	Use place value and number facts to solve problems involving all of the above and with increasingly large positive numbers	Use place value and number facts to solve problems involving all of the above	Use place value and number facts to solve problems involving all of the above	Use place value and number facts to solve problems involving all of the above

	Reception Year	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	Addition, Subtraction, Multiplication and Division						
Number Bonds	Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts	Develop fluency in addition and subtraction facts within 10. Compose numbers to 10 from 2 parts, and partition numbers to 10 into parts, including recognising odd and even numbers. Represent, memorise, use and reason with number bonds and related subtraction facts within 20 (e.g. 9 + 7 = 16; 16 – 7 = 9; 7 = 16 – 9)	Secure fluency in addition and subtraction facts within 10, through continued practice. Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 (e.g. using 3 + 7 = 10; 10 – 7 = 3 and 7 = 10 – 3 to calculate 30 + 70 = 100; 100 – 70 = 30 and 70 = 100 – 30)	Secure fluency in addition and subtraction facts that bridge 10, through continued practice.			
Addition and Subtraction Mental Methods and Understanding		Add and subtract one-digit and two-digit numbers to 20, including zero Read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs and relate additive expressions and equations to real-life contexts	Add/subtract across 10. Add and subtract numbers mentally, including: • a 2-digit number & 1s • a 2-digit number & 10s • two 2-digit numbers • adding three 1-digit numbers Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot Recognise the subtraction structure of 'difference' and answer questions of the form, "How many more…?".	Calculate complements to 100. Apply place-value knowledge to known additive number facts (scaling facts by 10). Add and subtract numbers mentally, including: • a three-digit number and ones • a three-digit number and tens • a three-digit number and hundreds Understand and use the commutative property of addition, and understand the related property for subtraction.	Apply place-value knowledge to known additive number facts (scaling facts by 100)	Apply place-value knowledge to known additive number facts (scaling facts by 1 tenth or 1 hundredth). Add and subtract numbers mentally with increasingly large numbers (e.g. 12,462 – 2,300 = 10,162)	Add and subtract numbers mentally with increasingly large numbers (e.g. 125,462 – 10,300 = 115,162) Use the compensation property of addition to complete equations such as , 25+ 35 = 27.5 +? and to help them solve calculations such as 27.5 + 32.5. Use their knowledge of the order of operations to carry out calculations involving the four operations (e.g. BIDMAS)

	Reception Year	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	Addition, Subtraction, Multiplication and Division						
Addition and Subtraction Written Calculations	Using quantities and objects, add and subtract two single-digit numbers and count on or back to find the answer	Read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs and relate additive expressions and equations to real-life contexts. *(also in mental calculation)*	Add and subtract numbers using concrete objects and pictorial representations, including: • a two-digit number and ones • a two-digit number and tens • two two-digit numbers • adding three one-digit numbers	Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction	Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate	Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)	
Inverse, Estimating, Rounding Answers and Checking			Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems (e.g. use addition to check subtractions)	Understand the inverse relationship between addition and subtraction, and how both relate to the part–part–whole structure. Estimate the answer to a calculation and use inverse operations to check answers	Estimate and use inverse operations to check answers to a calculation	Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy; round answers to a specified degree of accuracy, for example, to the nearest 10, 20, 50 etc, but not to a specified number of significant figures	Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy

	Reception Year	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	Addition, Subtraction, Multiplication and Division						
Multiplication and Division Facts and Mental Methods		Count forwards and backwards in multiples of 2, 5 and 10, up to 10 multiples, beginning with any multiple, and count forwards and backwards through the odd numbers.	Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot, and use commutativity and inverse relations to develop multiplicative reasoning (for example, 4 × 5 = 20 and 20 ÷ 5 = 4).	Continue to develop fluency in recalling multiplication facts, and corresponding division facts, in the 2, 5 and 10 multiplication tables. Recognise products in these multiplication tables as multiples of the corresponding number Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables (e.g. using 3 × 2 = 6, 6 ÷ 3 = 2 and 2 = 6 ÷ 3 to derive related facts - 30 × 2 = 60, 60 ÷ 3 = 20 and 20 = 60 ÷ 3) Recognise products in these multiplication tables as multiples of the corresponding number	Recall multiplication and division facts for multiplication tables up to 12 × 12, and recognise products in multiplication tables as multiples of the corresponding number. Combine their knowledge of number facts and rules of arithmetic to solve mental and written calculations (e.g. 2 x 6 x 5 = 10 x 6 = 60) Recognise and use factor pairs and commutativity in mental calculations (e.g. 4 × 12 × 5 = 4 × 5 × 12 = 20 × 12 = 240) Use place value, known and derived facts to multiply and divide mentally (e.g. 600 ÷ 3 = 200 can be derived from 2 x 3 = 6), including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers Manipulate multiplication and division equations, and understand and apply the commutative property of multiplication.	Secure fluency in multiplication table facts, and corresponding division facts, through continued practice. Multiply and divide numbers mentally drawing upon known facts, applying all the multiplication tables and related division facts frequently, commit them to memory and use them confidently to make larger calculations Apply place-value knowledge to known multiplicative number facts (scaling facts by 1 tenth or 1 hundredth). Multiply and divide numbers by 10, 100 and 1000; understand this as equivalent to making a number 10 , 100 or 1000 times the size, or 1 tenth,1 hundredth or 1 thousandth times the size. .	Continue to use all the multiplication tables to calculate mathematical statements in order to maintain fluency Use their knowledge of the order of operations to carry out calculations involving the four operations [e.g. explore the order of operations using brackets - 2 + 1 x 3 = 5 and (2 + 1) x 3 = 9]

					Understand and apply the distributive property of multiplication. Multiply and divide whole numbers by 10 and 100 (keeping to whole number quotients); understand this as equivalent to making a number 10 or 100 times the size.		

	Reception Year	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	Addition, Subtraction, Multiplication and Division *continued…*						
Properties of Numbers; Multiples, Factors, Squares, Cubes and Prime Numbers						Identify multiples and factors, including finding all factor pairs of a number and common factors and common multiples of two numbers. Express a given number as a product of 2 or 3 factors Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers Establish whether a number up to 100 is prime and recall prime numbers up to 19 Recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3})	Identify common factors, common multiples and prime numbers (including relating common factors to finding equivalent fractions)

	Reception Year	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	Addition, Subtraction, Multiplication and Division *continued…*						
Multiplication and Division Written Calculations			Write and calculate mathematical statements for multiplication and division within the multiplication tables and write them using the correct signs, working with a range of materials and contexts to relate to grouping and sharing discrete and continuous quantities, to arrays and to repeated addition Relate grouping problems where the number of groups is unknown to multiplication equations with a missing factor, and to division equations (quotitive division).	Write and calculate mathematical statements for multiplication and division using the multiplication tables that pupils know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods of short multiplication and short division	Multiply two-digit and three-digit numbers by a one-digit number using formal written layout Divide numbers up to 3 digits by a one-digit number using the formal written method of short division, without remainders	Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context, including with remainders, as fractions, as decimals or by rounding (for example, 98 ÷ 4 = 98/4 = 24 r2 = 24 1/2 = 24.5 ≈ 25). Multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000	Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication Divide numbers up to 4 digits by a two-digit whole number using the formal written methods of short and long division as appropriate, and interpret remainders as whole number remainders, fractions, decimals up to 2 decimal places, or by rounding, as appropriate for the context

	Reception Year	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	Addition, Subtraction, Multiplication and Division *continued…*						
Problem Solving	Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.	Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 =_ – 9) Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher (including doubling, halving and quartering numbers and quantities)	Solve problems with addition and subtraction: • using concrete objects and pictorial representations, including those involving numbers, quantities and measures • applying their increasing knowledge of mental and written methods Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts	Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction Solve problems, including missing number problems, involving multiplication and division, including integer scaling problems (e.g. four times as high, eight times as long) and correspondence problems in which n objects are connected to m objects (e.g. 3 hats and 4 coats, how many different outfits?) Apply known multiplication and division facts to solve contextual problems with different structures, including quotitive and partitive division.	Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why Solve problems involving multiplying and adding, including using the distributive law to multiply two-digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects Solve division problems, with two-digit dividends and one-digit divisors, that involve remainders, and interpret remainders appropriately according to the context.	Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why Solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates Use a given additive or multiplicative calculation to derive or complete a related calculation, using arithmetic properties, inverse relationships, and place-value understanding.	

	Reception Year	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	Fractions (including decimals and percentages)						
Counting, Recognising, Finding, Naming and Writing Fractions		Recognise, find and name a half as one of two equal parts of an object, shape or quantity (and understand it as equal grouping or sharing or objects or measures) Recognise, find and name a quarter as one of four equal parts of an object, shape or quantity (and understand it as equal grouping or sharing of objects or measures) Combine halves and quarters to make a whole	Count in fractions up to 10, starting from any number and using the 1/2 and 2/4 equivalence on the number line Recognise, find, name and write fractions 1/3, 1/4, 2/4 and 3/4 of a length, shape, set of objects or quantity, connecting unit fractions to equal sharing or grouping Write simple fractions 0f amounts (e.g. 1/2 of 6 = 3)	Interpret and write proper fractions to represent 1 or several parts of a whole that is divided into equal parts. Reason about the location of any fraction within 1 in the linear number system. Count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts (division) and in dividing one-digit numbers/quantities by 10; connect tenths to place value Recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators using known division facts (multiplication tables fluency). Recognise and use unit and non-unit fractions with small denominators as numbers (e.g. place fractions on number lines, going beyond 1 and relating to measures)	Count up and down in hundredths; recognise that hundredths arise when dividing an object by a hundred and dividing tenths by ten Understand the relation between non-unit fractions and multiplication and division of quantities, with particular emphasis on tenths and hundredths (e.g. 3/10 of 20 – multiply by 3 and divide by 10) Reason about the location of mixed numbers in the linear number system.	Continue to practise counting forwards and backwards in simple fractions, including bridging 0 (e.g. on a number line) Recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents Continue to develop understanding of fractions as numbers, measures and operators by finding fractions of numbers and quantities (e.g. 4/5 of 50)	

	Reception Year	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	Fractions (including decimals and percentages)						
Equivalence (including Fractions, Decimals and Percentages)			Recognise the equivalence of 2/4 and 1/2	Recognise and show, using diagrams, equivalent fractions with small denominators	Recognise and show, using diagrams, families of common equivalent fractions; use factors and multiples to recognise equivalent fractions and simplify where appropriate (e.g. 6/9 = 2/3) Recognise and write decimal equivalents to 1/4, 1/2, 3/4 Recognise and write decimal equivalents of any number of tenths or hundredths Convert mixed numbers to improper fractions and vice versa.	Identify name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths and understand that they have the same value and the same position in the linear number system. Recall decimal fraction equivalents for ½ , ¼ , 1/5and 1/10, and for multiples of these proper fractions. Read and write decimal numbers as fractions [e.g. 0.71 = 71/100] Recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents Recognise the per cent symbol (%) and understand that per cent relates to 'number of parts per hundred'; write percentages as a fraction with denominator hundred, and as a decimal	Use common factors to simplify fractions; use common multiples to express fractions in the same denomination (e.g. convert 3/4 and 2/5 into twentieths) Associate a fraction with division to calculate decimal fraction equivalents (e.g. 0.375) for a simple fraction[e.g. 3/8] Recall and use equivalences between simple fractions, decimals and percentages, including in different contexts (e.g. 3 ÷ 8 = 0.375)

	Reception Year	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	Fractions (including decimals and percentages) *continued…*						
Compare and Order Fractions				Compare and order unit fractions and fractions with the same denominators	Compare and order fractions with the same denominator, including tenths and hundredths, beyond one whole	Compare and order fractions whose denominators are all multiples of the same number	Express fractions in a common denomination and use this to compare and order fractions that are similar in value including fractions >1(e.g. order 1/2, 3/4, 2/3 and 13/12) Compare fractions with different denominators, including fractions greater than 1, using reasoning, and choose between reasoning and common denomination as a comparison strategy. for example, because 1/5 is greater 1/6 than we know that 2/5 is greater than 2/6 .
Adding and Subtracting Fractions				Add and subtract fractions with the same denominator within one whole [e.g. 5/7 + 1/7 = 6/7] Begin to add and subtract improper and mixed fractions with the same denominator, including bridging whole numbers.	Add and subtract fractions with the same denominator beyond one whole [e.g. 3/7 + 5/7 = 8/7]	Add and subtract fractions with the same denominator and denominators that are multiples of the same number, including calculations that exceed 1 as a mixed number (e.g. 2/3 + 5/6 = 4/6 + 5/6 = 9/6 = 1 3/6) Recognise mixed numbers and improper fractions >1 and convert from one form to the other; write mathematical statements >1 as a mixed number (e.g. 2/5 + 4/5 = 6/5 = 1 and 1/5)	Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions (e.g. 1 ½ and 4/5 = 30/20 + 16/20 = 46/20 = 1 and 16/20)

	Reception Year	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	Fractions (including decimals and percentages) *continued…*						
Multiplying and Dividing Fractions				Understand the relation between unit fractions as operators (fractions of), and division by integers.		Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams (e.g. 2/6 x 4 = 8/6 = 1 2/6)	Multiply simple pairs of proper fractions, writing the answer in its simplest form (e.g. 1/4 × 1/2 = 1/8) Divide proper fractions by whole numbers (e.g. 1/3 ÷ 2 = 1/6) Use understanding of the relationship between unit fractions and division to work backwards by multiplying a quantity that represents a unit fraction to find the whole quantity (for example, if 1/4 of a length is 36cm, then the whole length is 36 × 4 = 144cm).
Decimal Rounding					Round decimals with one decimal place to the nearest whole number	Round decimals with two decimal places to the nearest whole number and to one decimal place	Solve problems which require answers to be rounded to specified degrees of accuracy Round and estimate to predict and check the order of magnitude of answers to decimal calculations

	Reception Year	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	Fractions (including decimals and percentages*) continued…*						
Understanding, Reading, Writing, Comparing and Ordering Decimals					Compare and order numbers with the same number of decimal places up to two decimal places Learn decimal notation and the language associated with it, including in the context of measurements, and relate decimal notation to division of whole numbers by 10 and 100	Read, write, order and compare numbers with up to three decimal places Recognise the place value of each digit in numbers with up to 2 decimal places, and compose and decompose numbers with up to 2 decimal places using standard and non-standard partitioning Know that: 10 tenths are equivalent to 1 one; 1 is 10 times the size of 0.1; 100 hundredths are equivalent to 1 one; 1 is 100 times the size of 0.01; 10 hundredths are equivalent to 1 tenth; and 0.1 is 10 times the size of 0.01.	Identify the value of each digit in numbers given to three decimal places (e.g. identify the '7' in 0.874 as 7 hundredths or 70 thousandths
Adding and Subtracting Decimals						Mentally add and subtract tenths, and one-digit whole numbers and tenths Add and subtract decimals, including a mix of whole numbers and decimals (e.g. 3 – 2.58), decimals with different numbers of decimal places (e.g. 0.247 + 5.7), and complements of 1 (e.g. 0.83 + 0.17 = 1)	Mentally add and subtract hundredths, and one-digit whole numbers and hundredths Increase fluency of adding and subtracting decimals, including a mix of whole numbers and decimals (e.g. 3 – 2.58), decimals with different numbers of decimal places (e.g. 0.247 + 5.7), and complements of 1 (e.g. 0.83 + 0.17 = 1)

	Reception Year	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	Fractions (including decimals and percentages*) continued…*						
Multiplying and Dividing with Decimals					Find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths	Multiply and divide numbers by 10 and 100, giving answers up to three decimal places, identifying the value of the digits in the answer as ones, tenths and hundredths and thousandths	Multiply and divide numbers by 10, 100 and 1,000 giving answers up to three decimal places Multiply and divide numbers with up to two decimal places by one- and two- digit whole numbers Use written division methods in cases where the answer has up to two decimal places
Fraction, Decimal and Percentage Problem Solving		Solve problems that involve all above fraction learning	Solve problems that involve all above fraction learning	Solve problems that involve all above fraction learning	Solve problems involving increasingly harder fractions to calculate quantities and fractions to divide quantities, including non-unit fractions where the answer is a whole number Solve simple measure and money problems involving fractions and decimals to two decimal places	Solve fraction and decimal problems involving numbers up to three decimal places solve problems that require knowing percentage and decimal equivalents of 1/2, 1/4, 1/5, 2/5, 4/5 and those fractions with a denominator of a multiple of 10 or 25	Solve fraction and decimal problems which require answers to be rounded to specified degrees of accuracy

	Reception Year	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	Ratio and Proportion						
Relative sizes, percentage comparison, scale factors and unequal sharing							Solve problems involving the relative sizes of two quantities, where missing values can be found by using integer multiplication and division facts (e.g. Rob has 4 apples for every 1 that Jo has. Rob has 12 apples. How many does Jo have? Solve problems involving the calculation of percentages (e.g. of measures such as 15% of 360) and the use of percentages for comparison Solve problem involving similar shapes where the scale factor is known or can be found Solve problems involving unequal sharing and grouping using knowledge of fractions and multiples (e.g. 'for every egg you need three spoonfuls of flour')

	Reception Year	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	Algebra						
Formulae, problems and variables		Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = ? -9	Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems	Solve problems, including missing number problems			Express missing number problems algebraically Use simple formulae (e.g. to find missing numbers, lengths, coordinates and angles) Generate, describe and generalise linear number sequences Find pairs of numbers that satisfy an equation with two unknowns Enumerate possibilities of combinations of two variables Solve problems with 2 unknowns.
		Although algebraic notation is not formally introduced until year 6, algebraic thinking starts earlier, as exemplified by missing number problems					

	Reception Year	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	Measurement						
Comparing Measures	Use everyday language to talk about size, weight, capacity, position, distance, time and money to compare quantities and objects Develop spatial reasoning skills across all areas of mathematics including shape, space and measures	Compare, describe and solve practical problems for: • lengths and heights [e.g. long / short, longer / shorter, tall / short, double / half] • mass / weight [e.g. heavy / light, heavier than, lighter than] • capacity and volume [e.g. full / empty, more than, less than, half, half full, quarter] • time [e.g. quicker, slower, earlier, later] (moving from non-standard units to common standard units)	Compare and order lengths, mass, volume / capacity and record the results using >, < and = (e.g. half as high, twice as long)	Compare lengths (m/ cm/mm), mass (kg/g) and volume/capacity (l/ml), including simple scaling by integers (e.g. a given quantity or measure is twice as long or five times as high)	Compare different measures, including money in pounds and pence		
Measuring and Estimating Measures		Measure and begin to record the following: • lengths and heights • mass / weight • capacity and volume • time (hours, minutes, seconds) *(beginning to use measuring tools such as a ruler, weighing scales and containers; moving from non-standard units to common standard units)*	Choose and use appropriate standard units to estimate and measure length / height in any direction (m / cm); mass (kg / g); temperature (°C); capacity (litres / ml) to the nearest appropriate unit using rulers, scales, thermometers and measuring vessels	Measure lengths (m/ cm/mm), mass (kg/g) and volume/capacity (l/ml), including using mixed units (e.g. 1kg and 200g) Divide 100 into 2, 4, 5 and 10 equal parts, and read scales/number lines marked in multiples of 100 with 2, 4, 5 and 10 equal parts.	Measure and record metric measures using decimal notation, including money in pounds and pence Estimate different measures, including money in pounds and pence Divide 1,000 into 2, 4, 5 and 10 equal parts, and read scales/number lines marked in multiples of 1,000 with 2, 4, 5 and 10 equal parts.	Divide 1 into 2, 4, 5 and 10 equal parts, and read scales/number lines marked in units of 1 with 2, 4, 5 and 10 equal parts.	Divide powers of 10, from 1 hundredth to 10 million, into 2, 4, 5 and 10 equal parts, and read scales/number lines with labelled intervals divided into 2, 4, 5 and 10 equal parts.

	Reception Year	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	Measurement						
Money Understanding and Recognition (also see problem solving)	Use everyday language to talk about money	Recognise and know the value of different denominations of coins and notes	Recognise and use symbols for pounds (£) and pence (p); combine amounts to make a particular value Find different combinations of coins that equal the same amounts of money	Fluently use pounds (£) and pence (p) notation and understand the concept of giving change.	Represent pounds and pence using decimal notation.		
Tell the Time		Tell the time to the hour and half past the hour and draw the hands on a clock face to show these times Recognise and use language relating to dates, including days of the week, weeks, months and years	Tell and write the time to five minutes, including quarter past / to the hour and draw the hands on a clock face to show these times	Tell and write the time from an analogue clock to the nearest minute, including using Roman numerals from I to XI and 12-hour/24-hour digital clocks Estimate and read time with increasing accuracy to the nearest minute; record and compare time in terms of seconds, minutes and hours; use vocabulary such as o'clock / a.m. / p.m., morning, afternoon, noon and midnight	Read, write and convert time between analogue and digital 12-and 24-hour clocks		
Comparing and Sequencing Events and Intervals	Use everyday language to talk about time	Sequence events in chronological order using language (e.g. before and after, next, first, today, yesterday, tomorrow, morning, afternoon and evening)	Compare and sequence intervals of time (e.g. what is longer – 40 minutes or half an hour?)	Compare durations of events (e.g. to calculate the time taken by two trains using a timetable and say which journey takes longer)			

	Reception Year	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	Measurement						
Problem Solving	Use everyday language to talk about size, weight, capacity, position, distance, time and money to solve problems		Solve simple problems in a practical context involving addition and subtraction of money of the same unit, including giving change	Add and subtract amounts of money to give change, using both pounds (£) and pence (p) in practical contexts Add and subtract lengths (m, cm, mm) mass (kg, g) and volumes (l/ml)	Calculate different measures, including money in pounds and pence Solve problems involving converting from hours to minutes; minutes to seconds; years to months; weeks to days (e.g. Sally is 7 years and 2 months old; Macey is 85 months old – who is the oldest?)	Use all four operations to solve problems involving measures [money] using decimal notation, including scaling Use all four operations to solve problems involving measure [e.g. length, mass, volume/capacity, time] using decimal notation, including scaling Solve problems involving converting between units of time (e.g. how many seconds is 2.5 minutes?)	Solve problems involving the calculation and conversion of units of measure, using decimal notation up to three decimal places where appropriate Using a number line, use, add and subtract positive and negative integers for measures such as temperature

	Reception Year	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	Measurement						
Conversion			Know the number of minutes in an hour and the number of hours in a day	Know the number of seconds in a minute and the number of days in each month, year and leap year	Convert between different units of measurement (e.g. kilometre to metre; hour to minute) Solve problems involving converting from: hours to minutes; minutes to seconds; years to months; and weeks to days	Convert between different units of metric measure (e.g. kilometre and metre; centimetre and metre; centimetre and millimetre; gram and kilogram; litre and millilitre) including using common decimals and fractions. Understand and use approximate equivalences between metric units and common imperial units such as inches, pounds and pints	Use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation of up to three decimal places Solve problems involving the calculation and conversion of units of measure, using decimal notation up to three decimal places where appropriate Convert between miles and kilometres (including with a graphical representation)

	Reception Year	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	Measurement						
Area and Perimeter				Measure the perimeter of simple 2–D shapes	Measure and calculate the perimeter of a rectilinear figure (including squares) in centimetres and metres Find the perimeter of regular and irregular polygons. Find the area of rectilinear shapes by counting squares, and relate area to arrays and multiplication	Measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres, including using the relations of perimeter or area to find unknown lengths Calculate and compare the area of rectangles (including squares), and including using standard units, square centimetres (cm2) and square metres (m2), including using the relations of perimeter or area to find unknown lengths and estimate the area of irregular shapes	Recognise that shapes with the same areas can have different perimeters and vice versa Calculate the area of parallelograms and triangles, including the use of formulae (relating the area of rectangles to parallelograms and triangles, for example, by dissection, and then calculate their areas) Recognise when it is possible to use the formulae for the area of shapes (e.g. for a triangle when the base and perpendicular height is known)
Volume						Estimate volume (e.g. using 1cm^3 blocks to build cuboids, including cubes) and capacity (e.g. using water)	Calculate, estimate and compare volume of cubes and cuboids using standard units, including centimetre cubed (cm^3) and cubic metres (m^3), and extending to other units [e.g. mm^3 and km^3] Recognise when it is possible to use the formulae for the volume of shapes (e.g. cuboids)

	Reception Year	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	Geometry – properties and shapes						
Identifying 2-D Shapes and Their Properties	Explore characteristics of everyday objects and shapes and use mathematical language to describe them	Recognise and name common 2-D shapes (e.g. rectangles including squares, circles and triangles) in different orientations and sizes, relating to everyday objects	Identify and describe the properties of 2-D shapes (e.g. quadrilaterals and polygons), including the number of sides and line symmetry in a vertical line Use precise language to describe the properties of 2D shapes, and compare shapes by reasoning about similarities and differences in properties.	Describe the properties of 2-D shapes using accurate language, including lengths of lines and acute and obtuse for angles greater or lesser than a right angle	Identify regular polygons, including equilateral triangles and squares, as those in which the side-lengths are equal and the angles are equal. Identify lines of symmetry in 2-D shapes presented in different orientations. Reflect shapes in a line of symmetry and complete a symmetric figure or pattern with respect to a specified line of symmetry.	Describe a range of geometric shapes, using conventional markings for parallel lines and right angles Use the properties of rectangles to deduce related facts and find missing lengths/angles Use the term diagonal and make conjectures about the angles formed between sides, and between diagonals and parallel sides, and other properties of quadrilaterals	Describe the properties of a range of geometric shapes and explain how unknown angles and lengths can be derived from known measurements Illustrate and name parts of circles, including radius, diameter and circumference and know that the diameter is twice the radius
Drawing and Constructing 2-D Shapes		Compose 2D shapes from smaller shapes to match an example, including manipulating shapes to place them in particular orientations.		Draw 2-D shapes (symmetrical and non-symmetrical polygons) Draw polygons by joining marked points, and identify parallel and perpendicular sides	Complete a simple symmetric figure with respect to a specific line of symmetry Draw polygons, specified by coordinates and translate within the first quadrant	Draw rectangles with sides accurate to the nearest mm	Draw, compose, and decompose shapes according to given properties, including dimensions, angles and area, , using conventional markings and labels for lines and angles and solve related problems
Comparing and Classifying 2-D Shapes			Compare and sort common 2-D shapes and everyday objects reasoning about similarities and differences in properties (e.g. sides and vertices).		Compare and classify geometric shapes, including quadrilaterals (e.g. parallelogram, rhombus, trapezium) and triangles (e.g. isosceles, equilateral, scalene) based on their properties and sizes	Compare and classify a range of geometric shapes, using conventional markings for parallel lines and right angles Distinguish between regular and irregular polygons based on reasoning about equal sides and angles	Compare and classify a range of geometric shapes based on their properties and sizes, using conventional markings for parallel lines and right angles

	Reception Year	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	Geometry – properties and shapes						
Identifying 3-D Shapes and Their Properties		Recognise and name common 3-D shapes (e.g. cuboids including cubes, pyramids and spheres) in different orientations and sizes, relating to everyday objects	Identify and describe the properties of 3-D shapes (e.g. cuboids, prisms and cones) including the number of edges, vertices and faces. Use precise language to compare shapes by reasoning about similarities and differences in properties Identify 2-D shapes on the surface of 3-D shapes, (e.g. a circle on a cylinder and a triangle on a pyramid)			Identify 3-D shapes including cubes and other cuboids, from 2-D representations	Identify and describe a range of 3-D shapes
Drawing and Constructing 3-D Shapes		Compose 3D shapes from smaller shapes to match an example, including manipulating shapes to place them in particular orientations		Make 3-D shapes using modelling materials; recognise 3-D shapes in different orientations and describe them (symmetrical and non-symmetrical polyhedra)			Build simple 3-D shapes, including making nets
Comparing and Classifying 3-D Shapes			Use precise language to describe the properties of common 3D shapes. Compare and sort common 3-D shapes and everyday objects, reasoning about similarities and differences in properties.				

	Reception Year	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	Geometry – properties and shapes *continued…*						
Angles				Recognise that angles are a property of shape or a description of a turn Identify right angles in 2D shapes presented in different orientations. Recognise that two right angles make a half-turn, three make three quarters of a turn and four a complete turn; identify whether angles are greater than or less than a right angle Identify horizontal, vertical lines and pairs of perpendicular and parallel lines	Identify acute and obtuse angles and compare and order angles up to two right angles by size	Know angles are measured in degrees: estimate and compare acute, obtuse and reflex angles Identify: • angles at a point and one whole turn (total 360^{o}) • angles at a point on a straight line and 1/2 a turn (total 180^{o}) • other multiples of 90^{o} …and use this understanding to deduce missing angles and solve angle problems Draw given angles and measure them in degrees (^{o})	Recognise angles where they meet at a point, are on a straight line, or are vertically opposite, and find missing angles Find unknown angles in any triangles, quadrilaterals and regular polygons

	Reception Year	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	Geometry – position and direction						
Patterns	Recognise, create and describe patterns Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally		Order and arrange combinations of mathematical objects in patterns and sequences				
Movement, Grids and Coordinates	Use everyday language to talk about position and distance Develop spatial reasoning skills across all areas of mathematics including shape, space and measures	Describe position, directions and movement, including half, quarter and three-quarter turns	Use mathematical vocabulary to describe position, direction and movement, including movement in a straight line and distinguishing between rotation as a turn and in terms of right angles for quarter, half and three-quarter turns (clockwise and anti-clockwise), and apply in a practical context (e.g. pupils themselves moving in turns, giving instructions to other pupils to do so, and programming robots using instructions given in right angles)		Describe movements between positions as translations of a given unit to the left / right and up / down Draw a pair of axes in one quadrant, with equal scales and integer labels Describe positions on a 2-D grid as co-ordinates in the first quadrant [e.g. (2,5)] Plot specified points and draw sides to complete a given polygon	Identify, describe, plot and read the position of a shape following a reflection (reflection should be in lines that are parallel to the axes) or translation in one quadrant grid, and know that the shape has not changed Draw a pair of axes in one quadrant, with equal scales and integer labels Describe positions on a 2-D grid as co-ordinates in the first quadrant [e.g. (2,5)]	Draw and translate simple shapes in four quadrants, and reflect them in the axes Draw a pair of axes in four quadrants, with equal scales and integer labels Describe positions on the full co-ordinate grid (all four quadrants) [e.g. (-5, -3) Draw and label quadrilaterals in four quadrants, predicting missing coordinates using the properties of shapes

	Reception Year	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	Statistics						
Interpreting, Constructing and Solving Problems with Charts, Graphs and Tables			Interpret and construct simple pictograms, tally charts, block diagrams and simple tables (e.g. construct pictograms using many-to-one correspondence in pictograms with simple ratios 2, 5, 10) Ask and answer simple questions by counting the number of objects in each category and sorting the categories by quantity Ask and answer questions about totalling and comparing categorical data	Interpret and present data using bar charts, pictograms and tables (including understanding and using simple scales (e.g. 2, 5, 10 units per cm) with increasing accuracy) Solve one-step and two-step questions (e.g. 'How many more?' and 'How many fewer?') using information presented in scaled bar charts, pictograms and tables	Interpret and present discrete and continuous data using appropriate graphical methods, including bar charts and time graphs (including using a greater range of scales) Solve comparison, sum and difference problems using information presented in bar charts, pictograms, tables and other graphs	Interpret and present discrete and continuous data using appropriate graphical methods, including bar charts and time graphs (including using a greater range of scales), and begin to decide which representations of data are most appropriate and why Complete, read and interpret information in tables, including timetables Solve comparison, sum and difference problems using information presented in a line graph	Interpret and construct pie charts (including deducing required angles of sectors by applying understanding of angles and fractions) and line graphs and use these to solve problems (including connecting conversion from kilometres to miles in measurement to its graphical representation)
Mean Average							Calculate and interpret the mean as an average; understand when finding the mean is appropriate

Developing Mathematical Vocabulary and Definitions

73. Language is essential to the development of mathematical skills. Without a firm grasp on the vocabulary surrounding the subject of mathematics, things can quickly become disjointed and confusing to young minds.

74. The following talking activities should be undertaken with pupils to support the development of mathematical vocabulary:

- Explaining – giving a clear and detailed account;
- Describing – putting observations and experiences into words;
- Categorising – classifying according to common characteristics;
- Making connections – between items or information;
- Interpreting – perceiving the significance of connections;
- Predicting – using available information to estimate outcomes;
- Comparing – observing similarities and differences between items and relationships;
- Contrasting – observing differences between items or relationships;
- Clarifying – making clear their understanding;
- Justifying – providing evidence to prove a point;
- Elaborating – developing and extending ideas;
- Planning – organising ideas, stating ways of proceeding;
- Raising new questions – children ask their own questions and present issues;
- Investigating – examining systematically in order to solve a problem;
- Evaluating – judging and assessing;
- Arguing/conceding a case – presenting opposing/supporting reasons to a statement;
- Reasoning – drawing conclusions from facts and evidence;
- Hypothesising – suggesting an explanation for a group of facts;
- Reciting – reading or chanting aloud;
- Recounting – sharing personal experience and findings;
- Retelling – sharing information learned;
- Summarising – providing a brief account of the main points.

75. In accordance with the sequencing of content in the progression map, the following mathematical vocabulary should be taught and used in each Key Stage. The year group in brackets denotes in which year group a specific word should be *introduced*. This list of definition is cumulative; once a word has been introduced, it should be used from that point on. Therefore, for example, mathematical vocabulary that first appears in key stage 1 is not repeated in the Key Stage 2 column, although it should, importantly, continue to be used.

Reception Year	Key Stage 1	Key Stage 2
General		
Describe: When the curriculum asks pupils to 'describe' a mathematical object, transformation or the features of a graph, or anything else of a mathematical nature, it is asking pupils to refine their skills to home in on the essential mathematical features and to describe these as accurately and as succinctly as possible (year R) **Hundred Square:** A 10 by 10 square grid numbered 1 to 100 (year R) **Number line:** A line where numbers are represented by points upon it (year R) **Number track**: A numbered track along which counters might be moved. The number in a region represents the number of single moves from the start (year R) **Pattern:** A systematic arrangement of numbers, shapes or other elements according to a rule (year R) **Sequence:** A series of numbers or other elements which follow a rule (year R)	**Compare:** In mathematics when two entities (objects, shapes, curves, equations etc.) are compared one is looking for points of similarity and points of difference as far as mathematical properties are concerned (year 1) **Consecutive:** Following in order. Consecutive numbers are adjacent in a count (year 2) **Counter example:** Where a general statement is offered, an example that clearly disproves it (year 2) **Diagram:** A picture, a geometric figure or a representation (year 1) **Fluency:** To be mathematically fluent one must have a mix of conceptual understanding, procedural fluency and knowledge of facts to enable you to tackle problems appropriate to your stage of development confidently, accurately and efficiently (year 1) **General statement:** A statement that applies correctly to all relevant cases. e.g. the sum of two odd numbers is an even number (year 2) **Generalise:** To formulate a general rule (year 2) **Property:** Any attribute. Example: One property of a square is that all its sides are equal (year 2) **Quantity:** Something that has a numerical value, for example: 5 bananas (year 1) **Relationship:** The way in which two or more things are connected (year 2) **Scale (noun):** A measuring device usually consisting of points on a line with equal intervals (year 2) **Symbol:** A letter, numeral or other mark that represents a number, an operation or mathematical idea (year 1)	**Correspondence Problems**: Those in which m objects are connected to n objects (for example, 3 hats and 4 coats, how many different outfits?; 12 sweets shared equally between 4 children; 4 cakes shared equally between 8 children) (year 4) **Decimal:** Relating to the base ten. Most commonly used synonymously with decimal fractions where the number of tenths, hundredth, thousandths, etc. are represented as digits following a decimal point. The decimal point is placed at the right of the ones column. Each column after the decimal point is a decimal place (year 4) **Degree of accuracy**: A measure of the precision of a calculation, or the representation of a quantity. A number may be recorded as accurate to a given number of decimal places, or rounded to the nearest integer, or to so many significant figures (year 4) **Interpret:** Draw out the key mathematical features of a graph, or a chain of reasoning, or a mathematical model, or the solutions of an equation, etc (year 3) **Interval**: All possible points in the closed continuous interval between 0 and 1 on the real number line, including the end points zero and 1. Commonly used in statistics to describe the steps between two numbered points in a graph or chart axis. **Proof:** Using mathematical reasoning in a series of logical steps to show that if one mathematical statement is true then another that follows from it must be true (year 3) **Representation:** The word 'representation' is used in the curriculum to refer to a particular form in which the mathematics is presented, so for example a quadratic function could be expressed algebraically or presented as a graph; a quadratic expression could be shown as two linear factors multiplied together or the multiplication could be expanded out; a probability distribution could be presented in a table or represented as a histogram, and so on. Very often, the use of an alternative representation can shed new light on a problem. An array is a useful representation for multiplication and division which helps to see the inverse relationship between the two. The Bar Model is a useful representation of for many numerical problems (year 3)

Reception Year	Key Stage 1	Key Stage 2
Number and Place Value		
Count: The act of assigning one number name to each of a set of objects (or sounds or movements) in order to determine how many objects there are. **First**: Comes before all others in time or position (year R) **Next:** Comes immediately after the present one in order (year R) **Last:** Comes after all others in time or order. **Second:** an ordinal number, after first (year R) **Zero:** Nought or nothing; zero is the only number that is neither positive nor negative (year R)	**Cardinal number**: A cardinal number denotes quantity, as opposed to position within a series. 1, 2 and 3 are cardinal numbers. 1st, 2nd & 3rd and are ordinal (year 1) **Decreasing:** Becoming smaller in value (year 1) **Digit:** One of the symbols of a number system most commonly the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. Examples: the number 29 is a 2-digit number; there are three digits in 2.95. The position or place of a digit in a number conveys its value (year 1) **Even number**: An integer that is divisible by 2 (year 1) **Increasing:** Becoming greater in value. Used in relation to number Sequences (year 1) **Numeral:** A symbol used to denote a number. The Arabic numerals 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 are used in the Hindu-Arabic system giving numbers in the form that is widely used today (year 1) **Odd number**: An integer that has a remainder of 1 when divided by 2 (year 1) **Ordinal number:** A term that describes a position within an ordered set. Example: first, second, third (year 1) **Partition:** To split a number into component parts. Example: 38 = 30 + 8 or 19 + 19 (year 1) **Place value:** The value of a digit that relates to its position or place in a number (year 1) **Rule:** A procedure for carrying out a process. In the context of patterns and sequences a rule, expressed in words or algebraically, summarises the pattern or sequence and can be used to extend it (year 1) **Sequence:** A succession of terms formed according to a rule. Example: 1, 4, 9, 16, 25 etc (year 2)	**Integer:** Any of the positive or negative whole numbers and zero. Example: …- 2, -1, 0, +1, +2 … The integers form an infinite set; there is no greatest or least integer (year 3) **Negative number:** A number less than zero (year 4) **Place holder:** In decimal notation, the zero numeral is used as a place holder to denote the absence of a particular power of 10 (year 3) **Positive number:** A number greater than zero. Where a point on a line is labelled 0 positive numbers are all those to the left of the zero and are read 'positive one, positive two, positive three' etc (year 4) **Power (of 10):** 100 (i.e. 10^2 or 10 x 10) is the second power of 10, 1000 (i.e.10^3 or 10 x 10 x 10) is the third power of 10 etc (year 5) **Prime factors**: The factors of a number that are prime. Example: 2 and 3 are the prime factors of 12 (12 = 2 x 2 x 3) (year 5) **Prime factor decomposition:** The process of expressing a number as the product of factors that are prime numbers. Example: 24 = 2 × 2 × 2 × 3 or 23 × 3. Every positive integer has a unique set of prime factors **Prime number:** A whole number greater than 1 that has exactly two factors, itself and 1. Examples: 2 (factors 2, 1), 3 (factors 3, 1). 51 is not prime (factors 51, 17, 3, 1) (year 5) **Roman numerals:** The Romans used the following capital letters to denote cardinal numbers: I for 1; V for 5; X for 10; L for 50; C for 100; D for 500; M for 1000. Multiples of one thousand are indicated by a bar over a letter, so for example V with a bar over it means 5000. Other numbers are constructed by forming the shortest sequence with this total, with the proviso that when a higher denomination follows a lower denomination the latter is subtracted from the former (year 4) **Round (verb)**: In the context of a number, express to a required degree of accuracy. Example: 543 rounded to the nearest 10 is 540 (year 4)

Reception Year	Key Stage 1	Key Stage 2
Addition, Subtraction, Multiplication and Division		
Addition: The binary operation of addition on the set of all real numbers that adds one number of the set to another in the set to form a third number which is also in the set. The result of the addition is called the sum or total. **Altogether:** in total (year R) **Difference:** In mathematics (as distinct from its everyday meaning), difference means the numerical difference between two numbers or sets of objects and is found by comparing the quantity of one set of objects with another (year R) **Double:** 1) To multiply by 2. Example: Double 13 is (13 × 2) = 26; 2) The number or quantity that is twice another. Example: 26 is double 13 (year R) **Equal:** Symbol: =, read as 'is equal to' or 'equals'. and meaning 'having the same value as'. Example: 7 – 2 = 4 + 1 since both expressions, 7 – 2 and 4 + 1 have the same value, 5.	**Array:** An ordered collection of counters, numbers etc. in rows and columns (year 1) **Calculate:** To compute or work out mathematically (year 2) **Commutative:** A property of addition and multiplication. It does not matter in which order the addends or factors are added or multiplied; the result will be the same (year 2) **Consecutive**: Following in order. 2, 3, 4, 5, 6 are consecutive numbers. 3, 6 and 9 are consecutive multiples of 3. (year 2) **Divide:** To carry out the operation of division (Year 1) **Dividend:** In division, the number that is divided (year 2) **Division:** An operation on numbers interpreted in a number of ways. Division can be sharing – the number to be divided is shared equally into the stated number of parts; or grouping – the number of groups of a given size is found. Division is the inverse operation to multiplication (year 1) **Exchange:** Change a number or expression for another of equal value. The process of exchange is used in some standard compact methods of calculation (year 2) **Facts:** Multiplication / division/ addition/ subtraction facts. The word 'fact' is related to the four operations and the instant recall of knowledge about the composition of a number. i.e. an addition fact for 20 could be 10+10; a subtraction fact for 20 could be 20-9=11. A multiplication fact for 20 could be 4 x 5 and a division fact for 20 could be 20÷5 = 4 (year 1) **Four operations:** Common shorthand for the four arithmetic operations of addition, subtraction, multiplication and division (year 2)	**Approximation:** A number or result that is not exact. In a practical situation an approximation is sufficiently close to the actual number for it to be useful (year 3) **Associative law**: No matter how the parts in an addition or multiplication equation are grouped, the answer will be the same - (6 + 3) + 2 = 11, 6 + (3 + 2) = 11 (year 3) **Brackets:** Symbols used to group numbers in arithmetic or letters and numbers in algebra and indicating certain operations as having priority (year 2) **Columnar Addition and Subtraction:** A formal method of setting out an addition or a subtraction in ordered columns with each column representing a decimal place value and ordered from right to left in increasing powers of 10 (year 3) **Common Factors:** A number which is a factor of two or more other numbers, for example 3 is a common factor of the numbers 9 and 30 (year 5). **Common multiple**: An integer which is a multiple of a given set of integers, e.g. 24 is a common multiple of 2, 3, 4, 6, 8 and 12 (year 5) **Commutative:** Addition and multiplication are commutative because pairs of numbers can we calculated in any order. For example, 2 + 3 = 3 + 2 and 2 x 3 = 3 x 2 (year 3) **Compliment (in addition):** In addition, a number and its complement have a given total. Example: When considering complements in 100, 67 has the complement 33, since 67 + 33 = 100 (year 3) **Cube number:** A number that can be expressed as the product of three equal integers. Example: 27 = 3 × 3 × 3. Consequently, 27 is a cube number; it . It is the cube of 3 or 3 cubed. This is written compactly as 27 = 33 , using index, or power, notation (year 5) **Divisibility:** The property of being divisible by a given number. Example: A test of divisibility by 9 checks if a number can be divided by 9 with no remainder (year 3) **Divisor:** The number by which another is divided. Example: In the calculation 30 ÷ 6 = 5, the divisor is 6. In this example, 30 is the dividend and 5 is the quotient (year 3) **Efficient Methods**: A means of calculation (which can be mental or written) that achieves a correct answer with as few steps as possible. In written calculations this often involves setting out calculations in a columnar layout (year 3) **Estimate:** A rough or approximate answer (year 3) **Factor:** When a number can be expressed as the product of two numbers, these are factors of the first.

Fewer: A lesser amount – used when counting discrete objects, i.e. countable objects such as, pens, teddies, counters, etc (year R)

Less: A smaller amount or not as much (year R)

Minus: A name for the symbol –, representing the operation of subtraction (year R)

More: A greater amount (year R)

Number bond: A pair of numbers with a particular total e.g. number bonds for ten are all pairs of whole numbers with the total 10 (year 1)

Plus: A name for the symbol +, representing the operation of addition (year R)

Share: One model for the process of division (year R)

Subtract: Carry out the process of subtraction (year R)

Subtraction: The inverse operation to addition. Finding the difference when comparing magnitude. Take away (year R)

Greater than: An inequality between numbers. The symbol used to represent greater than is an arrow pointing towards the smallest number (year 2)

Inequality: When one number, or quantity, is not equal to another. Statements such as $a \neq b$, a b or $a \geq b$ are inequalities (year 2)

Inverse operations: Operations that, when they are combined, leave the entity on which they operate unchanged. Examples: addition and subtraction are inverse operations e.g. $5 + 6 - 6 = 5$. Multiplication and division are inverse operations e.g. $6 \times 10 \div 10 = 6$ (year 2)

Less than: An inequality between numbers. The symbol used to represent less than is an arrow pointing towards the smallest number (year 2)

Mental calculation: Referring to calculations that are largely carried out mentally, but may be supported with a few simple written jottings (year 1)

Missing number problems: A problem of the type $7 = \square - 9$ often used as an introduction to algebra (year 1)

Multiple: for any whole number, another number is a multiple if it is the answer to a multiplication question with the first number (e.g. 14, 49 and 70 are all multiples of 7) (year 2)

Multiplication: Multiplication (often denoted by the symbol "×") is the mathematical operation of scaling one number by another. It is one of the four binary operations in arithmetic (the others being addition, subtraction and division) (year 2)

Multiply: Carry out the process of multiplication (year 2)

Number sentence: A mathematical sentence involving numbers. Examples: $3 + 6 = 9$ and $9 > 3$ (year 1)

Examples: 1, 2, 3, 4, 6 and 12 are all factors of 12 because $12 = 1 \times 12 = 2 \times 6 = 3 \times 4$ (year 5)

Factorise: To express a number as the product of its factors. Examples: Factorising 12: $12 = 1 \times 12 = 2 \times 6 = 3 \times 4$ (year 5)

Formal written methods: Setting out working in columnar form (year 3)

Level of accuracy: Often in reference to the number of significant figures with which a numerical quantity is recorded, and made more precise by stating the range of possible error. The degree of precision in the measurement of a quantity (year 6)

Long Division: A columnar method for division by more than a single digit (year 6)

Long Multiplication: A columnar method for performing multiplication by more than a single digit (year 4)

Near double: See double (year 3)

Order of operation: This refers to the order in which different mathematical operations are applied in a calculation (year 6)

Product: The result of multiplying one number by another. Example: The product of 2 and 3 is 6 since $2 \times 3 = 6$ (year 3)

Quotient: The result of a division. Example: $46 \div 3 = 15\frac{1}{3}$ and $15\frac{1}{3}$ is the quotient of 46 by 3 (year 3)

Remainder: In the context of division requiring a whole number answer (quotient), the amount remaining after the operation. Example: 29 divided by 7 = 4 remainder 1 (year 5)

Short division: A compact written method of division (year 3)

Short multiplication: Essentially, simple multiplication by a one digit number, with the working set out in columns (year 3)

Square (multiplication): the square of a number is the product of the number and itself (year 5)

Square number: A number that can be expressed as the product of two equal numbers. Example $36 = 6 \times 6$ and so 36 is a square number or "6 squared". A square number can be represented by dots in a square array (year 5)

Sum: The result of one or more additions (year R) **Take away**: Subtraction as reduction (year R) **Total:** The sum found by adding (year R)	**Repeated addition:** The process of repeatedly adding the same number or amount. One model for multiplication. Example 5 + 5 + 5 + 5 = 5 x 4. (year 2) **Repeated subtraction:** The process of repeatedly subtracting the same number or amount. One model for division. Example 35 -5 - 5 - 5 - 5 - 5 - 5 - 5 = 0 so 35 ÷ 5 = 7 remainder 0 (year 2) **Sign:** A symbol used to denote an operation. Examples: addition sign +, subtraction sign −, multiplication sign ×, division sign ÷, equals sign = etc. In the case of directed numbers, the positive + or negative − sign indicates the direction in which the number is located from the origin along the number line (year 1)	

Reception Year	Key Stage 1	Key Stage 2
Fractions (including decimals and percentages)		
Half: One of two equal parts of a shape, quantity or object (year R)	**Denominator:** In the notation of common fractions, the number written below the line i.e. the divisor (year 2) **Equivalent fractions:** Fractions with the same value as another. For example: 4/8, 5/10, 8/16 are all equivalent fractions and all are equal to ½ (year 2) **Fraction:** The result of dividing one integer by a second integer, which must be non- zero. The dividend is the numerator and the non-zero divisor is the denominator (year 1) **Half:** one of two equal parts of a number, shape or quantity (year 1) **Non-unit fraction**: A fraction with a numerator greater than one (year 2) **Numerator:** In the notation of common fractions, the number written on the top – the dividend (the part that is divided). In the fraction ⅔, the numerator is 2 (year 2) **Quarter:** One of four equal parts of a whole, quantity or object (year 1) **Simple fraction:** A fraction where the numerator and denominator are both integers. Also known as common fraction or vulgar fraction (year 2) **Unit fraction:** A fraction that has 1 as the numerator and whose denominator is a non-zero integer. Example: ½, ⅓ (year 2)	**Cancel (a fraction):** One way to simplify a fraction down to its lowest terms. The numerator and denominator are divided by the same number e.g. 4/8 = 2/4. Also to 'reduce' a fraction (year 4) **Common fraction:** A fraction where the numerator and denominator are both integers. Also known as simple or vulgar fraction (year 3) **Decimal fraction:** Tenths, hundredths, thousandths etc represented by digits following a decimal point (year 4) **Improper fraction:** An improper fraction has a numerator that is greater than its denominator (year 4) **Mixed Fraction:** A whole number and a fractional part expressed as a common fraction. Example: 1⅓ is a mixed fraction. Also known as a mixed number (year 5) **Mixed Number:** A whole number and a fractional part expressed as a common fraction. Example: 2 ¼ is a mixed number. Also known as a mixed fraction (year 5) **Percentage:** A fraction expressed as the number of parts per hundred and recorded using the notation % (year 5) **Proper fraction:** A proper fraction has a numerator that is less than its denominator So ¾ is a proper fraction, whereas 4 /3 is an improper fraction (i.e. not proper) (year 3) **Recurring decimal:** A decimal fraction with an infinitely repeating digit or group of digits. Example: The fraction ⅓ is the decimal 0.33333 …, referred to as nought point three recurring and may be written as 0.3 (with a dot over the three). Where a block of numbers is repeated indefinitely, a dot is written over the first and last digit in the block e.g. 1 /7 = 0.˙142857˙ (year 6) **Simplify:** reduce a fraction to its simplest form (year 4) **Vulgar fraction**: A fraction in which the numerator and denominator are both integers. Also known as common fraction or simple fraction (year 3)

Reception Year	Key Stage 1	Key Stage 2
		Ratio and Proportion
		Proportion: A part to whole comparison. Example: Where £20 is shared between two people in the ratio 3 : 5, the first receives £7.50 which is 3 /8 of the whole £20. This is his proportion of the whole (year 6) **Ratio:** A part to part comparison. The ratio of a to b is usually written a : b. Example: In a recipe for pastry fat and flour are mixed in the ratio 1 : 2 which means that the fat used has half the mass of the flour, that is amount of fat/amount of flour = ½. Thus ratios are equivalent to particular fractional parts (year 6) **Scale:** To enlarge or reduce a number, quantity or measurement by a given amount (called a scale factor). e.g. to have 3 times the number of people in a room than before; to find a quarter of a length of ribbon; to find 75% of a sum of money (year 6) **Scale factor**: For two similar geometric figures, the ratio of corresponding edge lengths (year 6)
		Algebra
		Algebra: The part of mathematics that deals with generalised arithmetic. Letters are used to denote variables and unknown numbers and to state general properties. Example: a(x + y) = ax + ay exemplifies a relationship that is true for any numbers a, x and y (year 5) **Equivalent expression:** A numerical or algebraic expression which is the same as the original expression, but is in a different form which might be more useful as a starting point to solve a particular problem. Example: 6 + 10x is equivalent to 2(3 + 5x) (year 6) **Formula:** An equation linking sets of physical variables. e.g. A=πr2 is the formula for the area of a circle (year 5)

Reception Year	Key Stage 1	Key Stage 2
Measurement		
Before: prior to (year R) **Clock:** a tool to measure time (year R) **Empty:** Containing nothing. Most commonly used in the context of measures (year R) **Full:** Contains/holds as much or as many as possible; has no empty space (year R) **Length:** A linear measurement (year R) **Long:** An adjective used to describe length (year R) **Pound (money):** Symbol £. A unit of money. £1.00 = 100 pence. £1 is commonly called a pound (year R) **Short**: An adjective used to describe length (year R) **Tall:** Measuring a specific distance from top to bottom; opposite of short (year R)	**Analogue Clock**: A clock usually with 12 equal divisions labelled 'clockwise' from the top 12, 1, 2, 3 and so on up to 11 to represent hours. Commonly, each of the twelve divisions is further subdivided into five equal parts providing sixty minor divisions to represent minutes. The clock has two hands that rotate about the centre. The minute hand completes one revolution in one hour, whilst the hour hand completes one revolution in 12 hours (year 1). **Capacity:** the volume of a material (typically liquid or air) held in a vessel or container. Note: the term 'volume' is used as a general measure of 3-dimensional space and cannot always be used as synonymously with capacity (year 1) **Centimetre (cm):** A unit of linear measure equivalent to one hundredth of a metre (year 2) **Chronological:** Relating to events that occur in a time ordered sequence (year 2) **Gram:** Symbol: g. The unit of mass equal to one thousandth of a kilogram (year 2) **Hour:** A unit of time. One twenty-fourth of a day. 1 hour = 60 minutes = 3600 (60 x 60) seconds (year 1) **Length:** The extent of a line segment between two points. Length is independent of the orientation of the line segment (year 1) **Litre:** Symbol: l. A metric unit used for measuring volume or capacity. A litre is equivalent to 1000 cm3 (year 2) **Mass:** A characteristic of a body, relating to the amount of matter within it. Mass differs from weight, the force with which a body is attracted towards the earth's centre (year 1)	**Area**: A measure of the size of any plane surface. Area is usually measured in square units e.g. square centimetres (cm^2), square metres (m^2) (year 4) **Convert:** Changing from one quantity or measurement to another (year 3) **Cubic centimetre:** Symbol: cm^3 . A unit of volume. The three-dimensional space equivalent to a cube with edge length 1cm (year 6) **Cubic metre:** Symbol: m^3 . A unit of volume. A three-dimensional space equivalent to a cube of edge length 1m (year 6) **Digital Clock**: A clock that displays the time as hours and minutes passed, usually since midnight. Example: four thirty in the afternoon is displayed as 16:30 (year 4) **Foot:** Symbol: ft. An imperial measure of length. 1 foot = 12 inches. 3 feet = 1 yard. 1 foot is approximately 30 cm (year 6) **Gallon:** Symbol: gal. An imperial measure of volume or capacity, equal to the volume occupied by ten pounds of distilled water. In the imperial system, 1 gallon = 4 quarts = 8 pints. One gallon is just over 4.5 litres (year 6) **Imperial unit:** A unit of measurement historically used in the United Kingdom and other English speaking countries. Units include inch, foot, yard, mile, acre, ounce, pound, stone, hundredweight, ton, pint, quart and gallon. Now largely replaced by metric units (year 5) **Inch:** Symbol: in. An imperial unit of length. 12 inches = 1 foot. 36 inches = 1 yard. Unit of area is square inch, in2 . Unit of volume is cubic inch, in3 . 1 inch is approximately 2.54 cm (year 5) **Kilogram:** Symbol: kg. The base unit of mass in the SI (Système International d'Unités). 1kg. = 1000g (year 3) **Kilometre:** Symbol: km. A unit of length in the SI (Système International d'Unités). The base unit of length in the system is the metre. 1km. = 1000m (year 3) **Metre:** Symbol: m. The base unit of length (year 3) **Metric unit:** Unit of measurement in the metric system. Metric units include metre, centimetre, millimetre, kilometre, gram, kilogram, litre and millilitre (year 5) **Mile:** An imperial measure of length. 1 mile = 1760 yards. 5 miles is approximately 8 kilometres (year 6) **Millilitre:** Symbol: ml. One thousandth of a litre (year 5)

	Measuring tool: These record numerical quantities of continuous variables (year 1) **Minute:** Unit of time. One-sixtieth of an hour. 1 minute = 60 seconds (year 1) **Scale**: An object used to measure mass (year 1) **Second:** A unit of time. One-sixtieth of a minute (year 1) **Temperature:** A measure of the hotness of a body, measured by a thermometer or other form of heat sensor (year 2) **Time:** 1. Progress from past, to present and to future 2. Time of day, in hours, minutes and seconds; clocks and associated vocabulary 3. Duration and associated vocabulary 4. Calendar time in days, weeks, months, years 5. Associated vocabulary such as later, earlier, sooner, when, interval of time, clock today, yesterday, tomorrow, days of the week, the 12 months of a year, morning, a.m., afternoon, p.m., noon, etc (year 1) **Volume:** A measure of three-dimensional space. Usually measured in cubic units; for example, cubic centimetres (cm^3) and cubic metres (m^3) (year 1) **Weight:** In everyday English weight is often confused with mass. In mathematics, and physics, the weight of a body is the force exerted on the body by the gravity of the earth, or any other gravitational body (year 1)	**Ounce:** Symbol: oz. An imperial unit of mass. In the imperial system, 16 ounces = 1 pound. 1 ounce is just over 28 grams (year 6) **Perimeter:** The length of the boundary of a closed figure (year 3) **Pint**: An imperial measure of volume applied to liquids or capacity. In the imperial system, 8 pints = 4 quarts = 1 gallon. 1 pint is just over 0.5 litres (year 5) **Pound (mass)**: Symbol: lb. An imperial unit of mass. In the imperial system, 14 lb = 1 stone. 1 lb is approximately 455 grams. 1 kilogram is approximately 2.2 lb (year 5) **Rate:** A measure of how quickly one quantity changes in comparison to another quantity. For example, speed is a measure of how distance travelled changes with time; the average speed of a moving object is the total distance travelled/ time taken to travel that distance (year 6) **Rectilinear:** Bounded by straight lines. A closed rectilinear shape is also a polygon. A rectilinear shape can be divided into rectangles and triangles for the purpose of calculating its area (year 4) **Square centimetre**: Symbol: cm^2 . A unit of area, a square measuring 1 cm by 1 cm. 10000 cm2 = 1 m^2 (year 4) **Square metre:** Symbol: m2 . A unit of area, a square measuring 1m by 1 m (year 4) **Yard:** Symbol: yd. An imperial measure of length. In relation to other imperial units of length, 1 yard = 3 feet = 36 inches. 1760yd. = 1 mile One yard is approximately 0.9 metres (year 6)

Reception Year	Key Stage 1	Key Stage 2
Geometry – properties of shapes		
Circle: common 2-D shape with one curved side (year R) **Corner:** A point where two or more lines or line segments meet. More correctly called vertex, vertices (plural). Examples: a rectangle has four corners or vertices (year R) **Cube:** In geometry, a three-dimensional figure with six identical, square faces. Adjoining edges and faces are at right angles (year R) **Curved surface:** The curved boundary of a 3-D solid, for example; the curved surface of a cylinder between the two circular ends, or the curved surface of a cone between its circular base and its vertex, or the surface of a sphere (year R) **Cuboid:** A three-dimensional figure with six rectangular faces (year R) **Cylinder**: A three-dimensional object whose uniform cross-section is a circle (year R)	**2-D/3-D:** Short for 2-dimensional and 3-dimensional. A figure is two-dimensional if it lies in a plane. A solid is three-dimensional and occupies space (year 1) **Axis of symmetry**: A line about which a geometrical figure, or shape, is symmetrical or about which a geometrical shape or figure is reflected in order to produce a symmetrical shape or picture. Reflective symmetry exists when for every point on one side of the line there is another point (its image) on the other side of the line which is the same perpendicular distance from the line as the initial point (year 2) **Base:** The line or face on which a shape is standing (year 2) **Octagon:** A polygon with eight sides (year 2) **Pentagon:** A polygon with five sides and five interior angles (year 2) **Polygon:** A closed plane figure bounded by straight lines. The name derives from many angles (year 1) **Prism:** A solid bounded by two congruent polygons that are parallel (the bases) and parallelograms (lateral faces) formed by joining the corresponding vertices of the polygons. Prisms are named according to the base e.g. triangular prism, quadrangular prism, pentagonal prism (year 2) **Pyramid:** A solid with a polygon as the base and one other vertex, the apex, in another plane. Each vertex of the base is joined to the apex by an edge. Other faces are triangles that meet at the apex. Pyramids are named according to the base: a triangular pyramid (which is also called a tetrahedron, having four faces), a square pyramid, a pentagonal pyramid etc (year 1) **Quadrilateral:** A polygon with four sides (year 2)	**Acute angle:** An angle between 0^o and 90^o (year 3) **Angle:** An angle is a measure of rotation and is often shown as the amount of rotation required to turn one line segment onto another where the two line segments meet at a point (year 3) **Angle at a point**: The complete angle all the way around a point is 360° (year 5) **Angle at a point on a straight line:** The sum of the angles at a point on a line is 180° (year 5) **Arc**: A portion of the circumference of a circle (year 6) **Centre:** The middle point for example of a line or a circle (year 3) **Circumference:** The distance around a circle (its perimeter) (year 6) **Composite shape:** A shape formed by combining two or more shapes (year 3) **Compasses:** A tool for creating curved lines, arcs and circles (year 6) **Cross-section:** In geometry, a section in which the plane that cuts a figure is at right angles to an axis of the figure. Example: In a cube, a square revealed when a plane cuts at right angles to a face (year 3) **Decagon:** A polygon with ten sides and ten angles (year 3) **Degree:** The most common unit of measurement for angle (year 5) **Diagonal**: A line segment joining any two non-adjacent vertices of a polygon (year 5) **Diameter:** Any of the chords of a circle or sphere that pass through the centre (year 6) **Dodecahedron:** A twelve sided polygon **Equilateral triangle:** A triangle where all sides are of equal length and angles are the same (year 4) **Exterior angle:** angle on the outside of a shape (year 5) **Horizontal:** Parallel to the horizon (year 3) **Heptagon**: A polygon with seven sides and seven edges (year 3) **Hexagon:** A polygon with six sides and six edges (year 3)

Edge: A line segment joining two vertices of a plane figure (2-D shape) and the intersection of two plane faces (in a 3-D shape). **Face:** One of the flat surfaces of a solid shape. Example: a cube has six faces; each face being a square (year 1) **Flat:** A level surface (year R) **Oblong:** sometimes used to describe a non-square rectangle – i.e. a rectangle where one dimension is greater than the other (year R) **Rectangle**: A parallelogram with an interior angle of 90°. Opposite sides are equal. If adjacent sides are also equal the rectangle is a square. If adjacent sides are not equal, the rectangle is sometimes referred to as an oblong (year R) **Side**: A line segment that forms part of the boundary of a figure. Also edge (year R) **Surface:** An outer boundary of a 3-D object (year R)	**Sphere:** A closed surface, in three-dimensional space, consisting of all the points that are a given distance from a fixed point, the centre. A hemisphere is a half-sphere (year 1) **Symmetry:** A plane figure has symmetry if it is invariant under a reflection - i.e. if the effect of the reflection is to produce an identical-looking figure in the same position (year 2) **Vertex:** The point at which two or more lines intersect (year 2) **Vertical:** At right angles to the horizontal plane (year 2)	**Interior angle**: angle on the inside of a shape (year 5) **Irregular**: In geometry, irregular is a term used to describe shapes that are not regular (see below) (year 3) **Isosceles:** Isosceles triangles have two equal sides (year 4) **Kite**: A quadrilateral with two pairs of equal, adjacent sides whose diagonals consequently intersect at right angles (year 4) **Net:** A plane figure composed of polygons which by folding and joining can form a polyhedron (year 6) **Obtuse angle:** An angle greater than 90^{o} but less than 180^{o} (year 3) **Octahedron:** A polyhedron with eight faces. A regular octahedron has faces that are equilateral triangles (year 3) **Parallel:** denotes two lines that are always equidistant (year 3) **Parallelogram:** A quadrilateral whose opposite sides are parallel and consequently equal in length (year 4) **Perpendicular:** A pair of line segments (or surfaces) can be described as perpendicular if they intersect at (or form) a right angle (year 3) **Polyhedron:** A closed solid figure bounded by surfaces (faces) that are polygonal. Its faces meet in line segments called its edges. Its edges meet at points called vertices (year 3) **Protractor:** An instrument for measuring angles (year 5) **Radius**: In relation to a circle, the distance from the centre to any point on the circle (year 6) **Reflection:** In 2-D, a transformation of the whole plane involving a mirror line or axis of symmetry in the plane (year 4) **Reflex angle**: greater than 180 degrees (year 5) **Regular**: Describing a polygon, having all sides equal and all internal angles equal (year 3) **Rhombus**: A parallelogram with all sides equal (year 4) **Right:** Used as an adjective, right-angled or erect (year 3) **Right angle:** One quarter of a complete turn. An angle of 90 degrees. An acute angle is less than one right angle. An obtuse angle is greater than one right angle but less than two. A reflex angle is greater than two right angle (year 3) **Scalene:** A scalene triangle has three unequal sides and three unequal angles (year 4)

Square: A quadrilateral with four equal sides and four right angles (year R) **Triangle:** A polygon with three sides. Adjective: triangular, having the form of a triangle (year R)		**Set square:** A drawing instrument for constructing parallel lines, perpendicular lines and certain angles (year 5) **Similar:** Similar shapes are those which have the same internal angles and where the side lengths are in the same ratio or proportion. Enlarging a shape by a scale factor (for example by doubling all side lengths) creates a similar shape **Tetrahedron:** A solid with four triangular faces. A regular tetrahedron has faces that are equilateral triangles (year 4) **Trapezium:** A quadrilateral with exactly one pair of sides parallel (year 4) **Vertically opposite angles:** The pair of equal angles between two intersecting straight lines. There are two such pairs of vertically opposite angles (year 6)

Reception Year	Key Stage 1	Key Stage 2
Geometry – position and direction		
Above: Used to describe a higher position than another object (year R) **Backwards**: a movement in the direction behind (year R) **Below:** Used to describe a lower position than another object (year R) **Between:** Indicates a position in relation to two other places or objects on either side (year R) **Direction:** The orientation of a line in space (year R) **Forwards:** a movement in the direction in front (year R)	**Anticlockwise:** In the opposite direction from the normal direction of travel of the hands of an analogue clock (year 2) **Clockwise:** In the direction in which the hands of an analogue clock travel (year 2) **Cone:** A 3-dimensionsl shape consisting of a circular base, one vertex and one curved edge (year 1) **Direction:** The orientation of a line in space. e.g. north, south, east, west; up, down, right, left are directions (year 1) **Half turn:** A 180 degree rotation (year 1) **Left:** indicating a direction (year 1) **Quarter turn:** A rotation through 90º, usually anticlockwise unless stated otherwise (year 1) **Right**: indicating a direction (year 1) **Rotation**: In 2-D, a transformation of the whole plane which turns about a fixed point, the centre of rotation (year 1) **Turn:** A rotation about a point: a quarter turn is a rotation of 90°. A half turn is a rotation of 180°, a whole turn is a rotation of 360° (year 1)	**Axis:** A fixed, reference line along which or from which distances or angles are taken (year 4) **Coordinates System:** A system used to define the position of a point in two- or three-dimensional space. Two axes at right angles to each other are used to define the position of a point in a plane. The usual conventions are to label the horizontal axis as the x-axis and the vertical axis as the y-axis with the origin at the intersection of the axes. The ordered pair of numbers (x, y) that defines the position of a point is the coordinate pair. The origin is the point (0,0); positive values of x are to the right of the origin and negative values to the left, positive values of y are above the origin and negative values below the origin. Each of the numbers is a coordinate (year 4) **Coordinate:** In geometry, a coordinate uniquely determines the position of a point in space on a grid, using an x-coordinate and a y-coordinate (year 4) **Grid:** A lattice created with two sets of parallel lines (year 4) **Intersect**: the point at which two lines meet (year 6) **Origin:** A fixed point from which measurements are taken. In coordinates, denoted as (0,0) (year 4) **Plot:** The process of marking points. Points are usually defined by coordinates and plotted with reference to a given coordinate system (year 4) **Quadrant:** One of the four regions into which a plane is divided by the x and y axes in the Cartesian coordinate system (year 5) **Translation:** A transformation in which every point of a body moves the same distance in the same direction. A transformation specified by a distance and direction (year 4)

Reception Year	Key Stage 1	Key Stage 2
	Statistics	
	Bar chart/graph: A format for representing statistical information (year 2) **Block graph**: A simple format for representing statistical information. One block = one observation (year 2) **Carroll diagram:** A sorting diagram in which numbers (or objects) are classified as having a certain property or not having that property (year 2) **Chart:** Another word for a table or graph (year 2) **Data:** Information of a quantitative nature consisting of counts or measurements. Initially data are nearly always counts. When they refer to measurements that are separate and can be counted, the data are discrete. When they refer to quantities such as length or capacity that are measured, the data are continuous (year 2) **Frequency:** The number of times an event occurs; or the number of individuals with a specific property (year 2) **Minimum value:** The least value (year 2) **Maximum value:** The largest value (year 2) **Pictogram:** A format for representing statistical information. Suitable pictures, symbols or icons are used to represent objects. For large numbers one symbol may represent a number of objects and a part symbol then represents a rough proportion of the number (year 2) **Set:** A well-defined collection of objects) (year 2) **Sort:** To classify a set of entities into categories (year 2) **Table:** An orderly arrangement of information, numbers or letters usually in rows and columns (year 1) **Tally:** Make marks to represent objects counted; usually by drawing vertical lines and crossing the fifth (year 2)	**Average**: Loosely an ordinary or typical value, however, a more precise mathematical definition is a measure of central tendency which represents and or summarises in some way a set of data (year 6) **Column**: A vertical arrangement for example, in a table the cells arranged vertically (year 3) **Continuous Data:** Data arising from measurements taken on a continuous variable (examples: lengths of caterpillars; weight of crisp packets) (year 4) **Graph**: A diagram showing a relationship between variables (year 3) **Line Graph:** A graph in which adjacent points are joined by straight-line segments (year 4) **Mean:** Often used synonymously with average. The mean (sometimes referred to as the arithmetic mean) of a set of discrete data is the sum of quantities divided by the number of quantities (year 6) **Pie chart:** Also known as pie graph. A form of presentation of statistical information. Within a circle, sectors like 'slices of a pie' represent the quantities involved. The frequency or amount of each quantity is proportional to the angle at the centre of the circle (year 6)

Regular Practice of Core Skills

76. Core declarative and procedural knowledge should be practiced regularly and cyclically, regardless of what unit is currently being taught. This regular practice is a fundamental element of Star's mathematics curriculum offer, and supports pupils to know more and remember more, by providing regular opportunities to recall and revisit core knowledge. For example, even though Year 2 may be undertaking a spring term unit that focusses on *Properties of Shape*, pupils should still be recalling and practising their 2, 5, and 10 times tables, as well the core Year 2 procedural knowledge of column addition and subtraction,
77. Regular declarative and procedural practice can occur in a number of ways, including:
    - Additional discrete arithmetic sessions.
    - Additional discrete times tables/core number skills sessions.
    - *Do It Now* activities at the starts of lessons.
    - A focus on number and calculation knowledge organisers during 'dead time'.
78. Discrete declarative and procedural knowledge sessions should focus on the following year group curriculum objectives (some of these objectives, such as those focussed on times table, are taken from earlier year groups because they are considered to be central to number fluency:

<u>Reception Year</u>

- Represent and use number bonds and related subtraction facts within 10;
- Add and subtract two single-digit numbers up to 20 and count on or back to find the answer.

<u>Year 1</u>

- Count, read and write numbers to 100 in numerals; count in multiples of 2s, 5s and 10s;
- Given a number, identify 1 more and 1 less;
- Represent and use number bonds and related subtraction facts within 20;
- Add and subtract one-digit and two-digit numbers to 20, including 0.

<u>Year 2</u>

- Count in steps of 2, 3, and 5 from 0, and in 10s from any number, forward and backward;
- Recognise the place value of each digit in a two-digit number (10s, 1s);
- Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100;
- Add and subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and 1s; a two-digit number and 10s; 2 two-digit numbers; and adding 3 one-digit numbers;
- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.

<u>Year 3</u>

- Count from 0 in multiples of 4, 8, 50 and 100; find 10 or 100 more or less than a given number;

- Recognise the place value of each digit in a 3-digit number (100s, 10s, 1s);
- Add and subtract numbers mentally, including: a three-digit number and 1s; a three-digit number and 10s; and a three-digit number and 100s;
- Add and subtract numbers with up to 3 digits, using formal written methods of columnar addition and subtraction;
- Recall and use multiplication and division facts for the 2, 3, 4, 5, 8 and 10 multiplication tables;
- Apply place-value knowledge to known additive and multiplicative number facts (scaling facts by 10);
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.

Year 4

- Count in multiples of 6, 7, 9, 25 and 1,000;
- Find 1,000 more or less than a given number;
- Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate;
- Recall multiplication and division facts for multiplication tables up to 12 × 12;
- Apply place-value knowledge to known additive and multiplicative number facts (scaling facts by 100)
- Multiply two-digit and three-digit numbers by a one-digit number using formal written layout;
- Divide numbers up to 3 digits by a one-digit number using the formal written method of short division, without remainders.

Year 5

- Count forwards or backwards in steps of powers of 10 for any given number up to 1,000,000;
- Add and subtract whole numbers with more than 4 digits including using formal written methods (columnar addition and subtraction);
- Add and subtract numbers mentally with increasingly large numbers;
- Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers;
- Multiply and divide numbers mentally drawing upon known facts;
- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context;
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000;
- Recall multiplication and division facts for multiplication tables up to 12 × 12.

Year 6

- Add and subtract whole numbers with more than 4 digits including using formal written methods (columnar addition and subtraction);
- Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication;

- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division;
- Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context;
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000;
- Recall multiplication and division facts for multiplication tables up to 12 × 12.

79. In order to support pupils to 'know more and remember more', related learning from prior lessons, units, terms and/or years is also revisited and embedded at the starts of lessons.

Maths Mastery

80. Teaching maths for mastery is a transformational approach to maths teaching which stems from high performing Asian nations such as Singapore. When taught to master maths, children develop their mathematical fluency without resorting to rote learning and are able to solve non-routine maths problems without having to memorise procedures.
81. Maths mastery is an evidence-based approach to teaching maths which helps pupils develop a deep, long-term and adaptable understanding of maths.
82. It is an inclusive approach where all children achieve. When teaching maths for mastery, the whole class moves through topics at broadly the same pace. Each topic is studied in depth and the teacher does not move to the next stage until all children demonstrate that they have a secure understanding of mathematical concepts. Thus, the vast majority of pupils work at the same pace, *on the same age-related content*, with different groups of pupils with different needs supported with additional scaffolds or moved on through the provision of deeper, richer and more complex questions, problems and puzzles. This inclusive approach, and its emphasis on promoting multiple methods of solving a problem, builds self-confidence and resilience in pupils, whilst both providing additional support that need it and challenging more able learners. A maths mastery approach ensures that all can master concepts before moving to the next part of the curriculum sequence, allowing no pupil to be left behind.
83. If a pupil fails to grasp a concept or procedure, the Star way is to identify this quickly, and early intervention ensures the pupil is ready to move forward with the rest of the class. The notion of 'catch up and keep up' is central to our approach to maths, and dovetails seamlessly with maths mastery.
84. Content is covered at a slower pace which results in greater progress. Pupils are given time to think deeply about the maths and really understand concepts at a relational level rather than as a set of rules or procedures.
85. Teaching maths for mastery is a key plank of the Government's education reforms and is reflected in the 2014 English national curriculum for mathematics. It is also endorsed by all popular mathematics schemes and frameworks, such as White Rose Maths, Maths No Problem, Power Maths and Inspire Maths. It is also the approach endorsed by the Department for Education, NCETM and OFSTED.
86. In Star maths mastery lessons:
    - New learning, the key points, and the **carefully sequenced journey** through the learning are clearly identified. Here, lessons are broken down into **small, connected steps** that gradually unfold the concept, providing access for all children and leading to a **generalisation of the concept** and the ability to apply the concept to a range of contexts;
    - The **CPA approach** underpins the approach (concrete, pictorial, abstract). **Physical and pictorial representations** used in lessons expose the mathematical structure being taught, the aim being

that most pupils progress to doing the maths without recourse to the representation. Concrete and pictorial representations of mathematics are chosen carefully to help build procedural and conceptual knowledge together;

- The principle of **variation** is employed. Variation is twofold. It is firstly about how the teacher represents the concept being taught, often in more than one way, to draw attention to critical aspects, and to develop deep and holistic understanding. It is also about the sequencing of the episodes, activities and exercises used within a lesson and follow up practice, paying attention to what is kept the same and what changes, to connect the mathematics and draw attention to mathematical relationships and structure;
- Pupils often sit facing the teacher and the teacher leads back and forth interaction, including questioning, short tasks, explanation, demonstration, and discussion;
- It is recognised that practice is a vital part of learning, but the practice used is intelligent practice that both reinforces pupils' procedural fluency and develops their conceptual understanding.
- Significant time is spent developing deep knowledge of the key ideas that are needed to underpin future learning. The structure and connections within the mathematics are emphasised, so that pupils develop deep learning that can be sustained;
- Key declarative knowledge such as multiplication tables and addition facts within 10 are learnt to automaticity to avoid cognitive overload in the working memory and enable pupils to focus on new concepts;
- Learning is started with an initial cyclical focus on key number and arithmetic facts and understanding, as per paragraph 62 above.

The Concrete Pictorial Abstract Approach

Concrete Representations

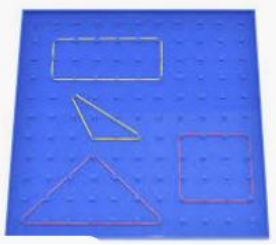
87. In Star schools, we know that our pupils learn best through a Concrete Pictorial Abstract approach (CPA). New learning is introduced using concrete and pictorial approaches, with rigorous support for children to move towards abstract methods. The initial concrete phase brings concepts to life, and can be characterised as the 'doing' stage. Manipulatives play a key role in this as they are the concrete resources used to support pupils thinking as they explore abstract ideas. Using something 'real' to make sense of the maths takes away the need to imagine or visualise at the early stage of learning a new concept. Concrete maths manipulatives provide the pupil a 'window' in, to make sense of the problem at hand by touching them, playing with them, exploring the patterns and relationships which make a huge difference between understanding for depth or just for procedure.
88. Time is taken to train and support teachers to use concrete objects correctly and effectively. Concrete objects are used where necessary, but it is also understood that after initial conceptual development has occurred, manipulatives are usually phased out, replaced by pictorial representations. This second stage of moving to the predominant use of pictorial resources can be characterised as the 'seeing' stage. Finally, pupils move to abstract methods – this can be characterised as the 'symbolic' stage. Here, concrete resources and pictorial representations are tools to supporting pupils to gain deep conceptual understanding, but should not be relied upon in the long term. However, it must be highlighted that at any point in a teaching sequence, pupils may move back and forth between concrete, pictorial and abstract representations, in order to support learning.
89. The amount of time pupils need to progress from concrete to abstract understanding varies. For some, it might occur within a single lesson, but for others, it will develop gradually over a teaching sequence.

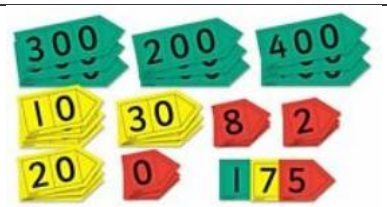
In each teaching sequence, pupils start with a concrete experience. As they move towards the pictorial, they might move between the pictorial and the concrete in order to clarify and refine their understanding, eventually reaching the abstract understanding.

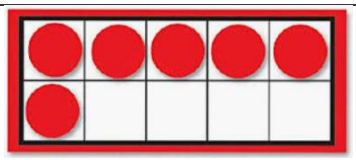
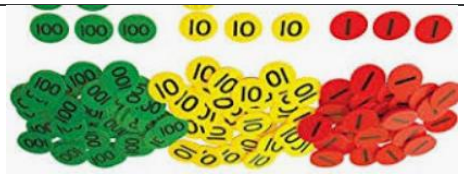
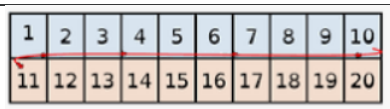
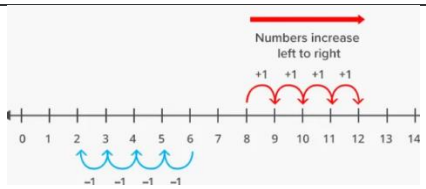
90. Reliance and subsequent dependence on manipulatives and associated aids can hinder progression through the curriculum. Teachers need to give pupils enough time to consolidate learning and they need to plan for how pupils will move away from using the manipulatives. This will help to avoid pupils relying on manipulatives to work around gaps in core knowledge that might become barriers to learning later.

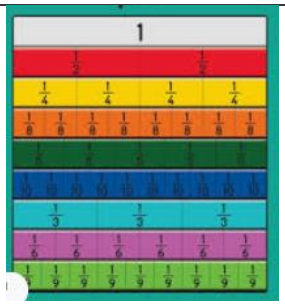
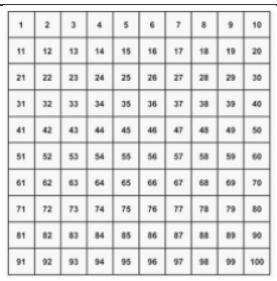
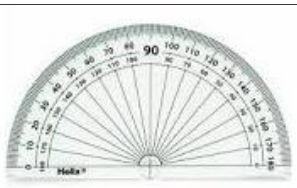
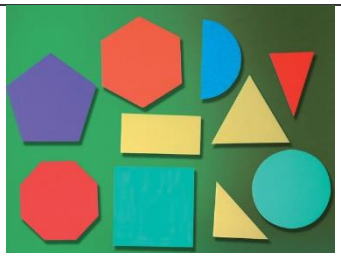
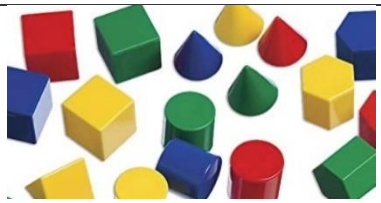
91. The concept of variation also applies to the use of concrete resources. Fundamental to maths mastery is the ability to use and apply concepts to a wide range of concepts by gaining deep transferable understanding of concepts. One step towards achieving this is through variation – variation of the problem posed, but also variation on the manipulatives used. Here, there is not just one concrete resource used to explore a concept and answer questions. For instance, when teaching number and place value, a range of manipulatives can be used, including: Numicon, bead strings, Dienes blocks, and place value counters. Pupils who require more support to master a specific concept often achieve success through the structured use of a wide variety of manipulatives.

92. Important concrete resources that should be used in Star schools to support deep conceptual development include:

Name of resource	Image	Description
Dienes Apparatus		Plastic blocks, in ones, rows of ten and larger arrays of 100 and 1000s. They support pupils in developing an understanding of place value, as well as exploring the concept of regrouping in addition and subtraction, and modelling the process for long division.
Numicon		Structured apparatus with different sized and coloured pieces representing each number. Numicon's imagery uses patterns to represent each numeral. The patterns are structured so number relationships can be seen and experienced. These can also be used to teach about fractions, decimals and percentages. Great resources for small group or one-to-one intervention. Numicon is also weighted so you can use it with scales.
Geoboards		A great way for exploring geometry. Rubber bands are stretched over the pegs to make different shapes, perfect for exploring sides and vertices, investigating area and perimeter, symmetry and angles. A variety of different geoboards are available from isometric boards to coordinate boards.

Place Value Arrow Cards		These are ideal for reinforcing place value and practising partitioning.
Dice		Dice are excellent for engaging pupils in maths activities. They are particularly suitable for getting pupils to carry out quick mental Maths tasks. Ten sided dice can also be used for supporting the exploration of place value concepts.

Name of resource	Image	Description
Bead strings		A class set of bead strings is an essential resource for classrooms. These consist of a short string with beads on, each alternate group of ten coloured differently. These are great for counting on and back in ones or ten, exploring number bonds to 10 or 100, and as a visual way of demonstrating fractions, decimals, percentages and the four operations. They are also an effective tool for looking at counting on and back, helping pupils understand number lines.
Cuisenaire Rods		Plastic or wooden rods are coloured coded depending on their size (from 1 unit to 10 units). They have a whole host of applications including number bonds, patterns, fractions, decimals, bar modelling, scaling and ratios.
Tens Frame		These support the central concept that a that single digits also be partitioned and used to aid mental addition (or subtraction). Tens frames can also be used to teach fractions and decimals.
Place Value Counters		These counters show place value and are invaluable for teaching all four operations – particularly for multiplication and division. The counters give a concrete representation of the number, especially important when working with larger numbers and decimal numbers.
Number tracks		These are often used with younger pupils as a precursor to using number lines, for developing early understanding of number, counting on and counting back – can be used in conjunction with counters.
Number lines		These are a very versatile resource and can be used to build understanding of all four operations.
Counters		Counters are excellent for developing basic maths skills such as counting, sorting and pattern making.

Name of resource	Image	Description
Interlocking cubes		These are very useful for developing patterns, both one- and two-dimensional, based on colour. When the cubes are used to build three-dimensional structures, they lead naturally to the concepts of volume and surface area. They are also very useful for young pupils developing a wide range of basic number concepts.
Fraction boards		These are very useful visual representation to help pupils learn, compare and identify fractions, set out in the form of a wall. They are excellent for supporting both fraction equivalence and arithmetic.
100-squares		These can be used to support the development of many number concepts, such as number patterns, odd and even numbers, basic counting and the introduction of basic addition and subtraction.
Protractors		Protractors are a key resource in upper key stage 2 for developing the skills of measuring and drawing angles.
2-D shapes		Excellent for sorting, comparing and discussing the properties of 2-D shapes.
3-D shapes		Excellent for sorting, comparing and discussing the properties of 3-D shapes.
Number balance		A Number Balance visually demonstrates equivalents to help pupils understand number relationships and operations, such as addition, subtraction, comparisons and algebra. Can be used with Numicon, which is proportionally weighted.

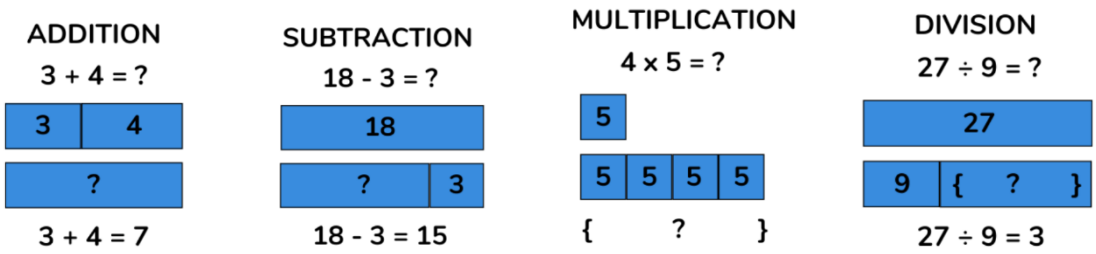
Name of resource	Image	Description
2 colour counters		Have a wide range of uses, including in conjunction with tens frames.
Digit cards		These are a very versatile resource for pupils to independently generate numbers when developing a wide range of number concepts.

Bar Modelling

93. Pictorial representations, the 'seeing' stage, are a key component of a pupil's transition from the use of concrete resources, when initially exploring a new concept, to gaining a deep conceptual understanding, where the related schema is internalised and only abstract 'symbolic' representation is required.
94. Bar models thus act as a 'bridge' between the concrete, pictorial and abstract (CPA in maths); once children are secure with using pictorial versions of their concrete materials, they can progress to using bars as visual representations.
95. The bar model is a pictorial representation of a problem or concept where bars or boxes are used to represent the known and unknown quantities. In word problems, bar models help children decide which operations to use and to visualise problems.
96. The bar model is integral to this pictorial approach when teaching number, and has a wide range of uses when modelling and exploring concepts across both key stage 1 and 2. Its value in supporting a deep understanding of underlying concepts is most apparent when tackling mastery questions that require reasoning and problem solving skills.
97. Bar models do not, however, do calculations for the pupil nor solve problems and reason by themselves; rather, they simply make it easier for pupils to understand the maths that is underpinning a question by providing a pictorial model, work out which calculation must be done and solve a problem accordingly.
98. Bar modelling has many varieties and can take on many forms. Different maths schemes and sources of bar modelling CPD for teachers all utilise slightly varied pictorial versions of the bar model, but the underlying principles are the same. It is essential here that the exact visual approach to bar modelling is agreed at a whole-school level, to support pupils' cohesive development in understanding these visual representations.
99. More substantial Trust guidance regarding the important role that bar modelling has in developing mathematical understanding is set out in the Trust document 'Bar Modelling Star Maths Guidance'.

Basic Calculations with the Four Operations

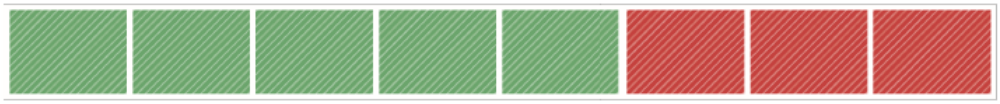
100. At the basic level, all four operations can easily be represented, modelled and explored by pupils with the support of a bar model, once concrete resources have provided a sufficient introduction to a concept:



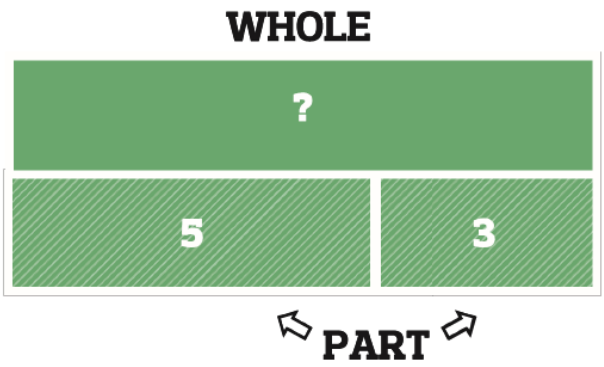
Progressing from the Concrete to the Bar Model

101. Initially, when younger pupils are set a problem, real objects may be used to represent a problem. An example of a simple addition problem is: 'There are 5 green apples and 3 red apples in the bag. How many apples are in the bag? Here, a bag of real apples is initially used . The teacher pulls the apples out the bag, sorts them into red and green ones, puts them into a line and supports the pupils to count them. Here, parts are put together to find the whole.

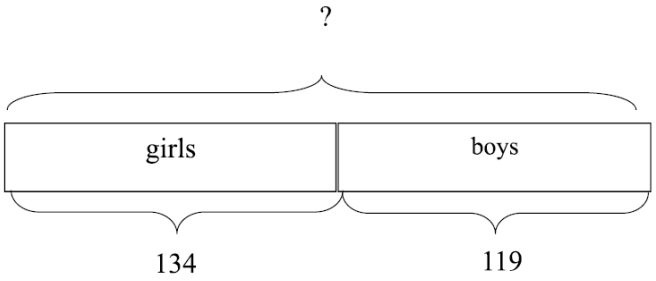
102. The next step in conceptual development, still remaining in the zone of concrete representation, is to model the problem using something else to represent the apples, such as green and red counters or cubes. Although this is still concrete, it is a generic representation. The objects can still be lined up and counted, but the objects could represent three and five of anything.
103. The representation of the problem now develops to using pictorial methods. One-to-one representation is still used, but now the objects are drawn. Actual apples could initially be drawn but pupils would usually settle comfortably on using simple picture such as a square or cross to represent each apple:



104. The teacher now progresses beyond one-to-one representation, using a proportional bar instead. Older pupils will usually grasp this conceptual leap quicker than younger pupils. One bar can be drawn to represent the whole, and the parts can be drawn underneath in a separate bar. More complex problems require different arrangements of bars, but the underlying principle is the same:

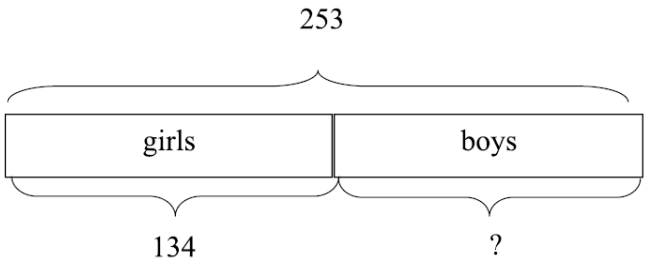


105. In these addition and subtraction 'part-whole models' the known value can be placed in the bars and the unknown value can be identified using a question mark.
106. In the above example, there was a smooth transition from the use of the actual real objects, to using representative objects, and then to pictorial one-to-one representation, and then finally, the proportional bar model. The CPA approach can thus be characterised as a continuum.
107. Part-whole bar models can easily be used to pictorially represent more challenging addition and subtraction problems, and with older pupils the progression to a proportion bar, which does not rely on one-to-one pictorial representation, is usually more rapid. For example, a question could be: '134 girls and 119 boys took part in an art competition. How many children took part in the art competition?':

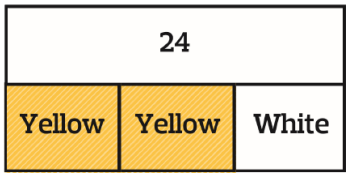


Here, known values are not written within bars, but brackets are used instead. Thus, the total is identified with a bracket rather than a separate bar. This is simply a matter of convention, according to which scheme/type of bar model format is being used in a particular school. It is recommended that a whole-school systematic approach is used here.

108. Subtraction problems can also use the part-whole bar model in a very similar way. For example, a question could be: '253 pupils took part in an art competition. 134 were girls. How many boys took part in the competition?':



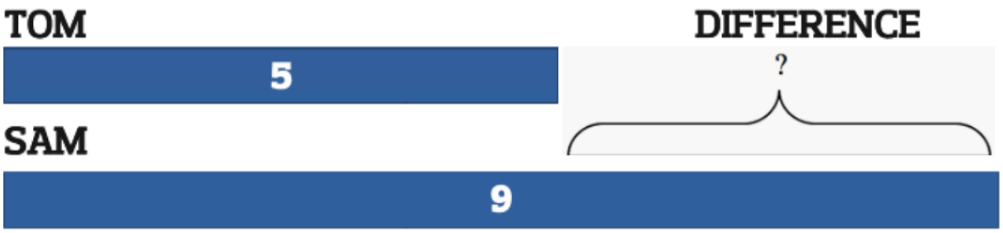
109. Multiplication and division can still be represented using 'part-whole models' because there are still knowns and unknowns and a part-whole relationship still exists, but the bar may need to be split into more parts to represent a problem:



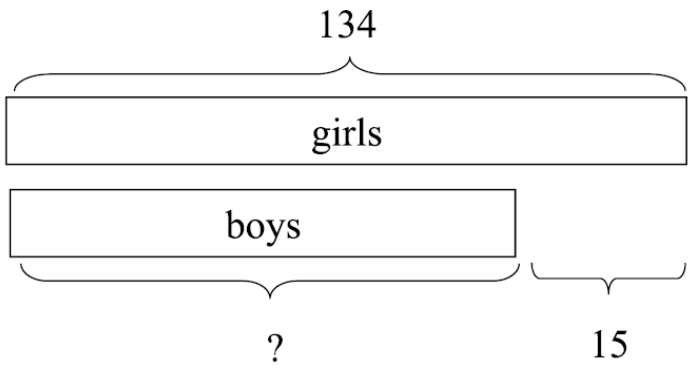
Note here that in this example, the convention now being used is the practice of employing a separate, second bar for the total value – 24.

110. Another type of bar model is the 'comparison model'. This allows us to compare two quantities, such as when pupils are finding the difference. An example question could be: 'Tom is playing with 5 toy cars and Mo is playing with 9 toy cars. How many more than Tom does Mo have?

111. Here, initially, as per the previous apple example, real objects and one-to-one pictorial representations are used. As the model is developed, a comparison bar model can be used. Here, two bars are used to represent the two quantities, and the unknown value can be represented with a question mark and bracket:



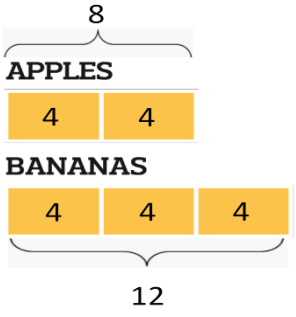
112. More challenging 'finding the difference' problems can also be tackled using the comparison model. For example, a question could be: 134 girls took part in an art competition. 15 fewer boys than girls took part. How many boys took part?' The boys are being compared to the girls, and we know the larger quantity:



113. This 'comparison model' is particularly useful for tackling ratio and proportion questions in year 6.

114. For example, the question could be: 'The ratio of apples to bananas in the fruit bowl is 2:3. If there are 12 bananas, how many apples are there?'

Here, one bar is used to represent the apples and one for the bananas. It is a comparison model because two quantities are being compared. The known ratio, 2:3, is used to draw the bars. The known information is added to the picture, and this allows the quantity of apples to be worked out:



115. Bar modelling is central to maths mastery; it provides the crucial step when developing problem solving and reasoning skills, from one-to-one pictorial representation to abstract methods.

Calculation Policy

116. Pupils should be introduced to procedural knowledge of calculation through practical, oral and mental activities. Development and practice of necessary age-related calculation skills, as per the 'addition, subtraction, multiplication and division' strands of the progression map, is undertaken both within specific medium term planning units, and in additional discrete opportunities, as per paragraph 75.

117. As pupils deepen their arithmetic understanding, they develop ways of recording to support their thinking and calculation methods, use particular methods that apply to special cases, and learn to interpret and use the signs and symbols involved. Choosing the appropriate strategy, recording in mathematics and in calculation in particular, is an important tool both for furthering the understanding of ideas and for communicating those ideas to others.

118. A useful written method is one that helps children carry out a calculation and can be understood by others. Written methods are complementary to mental methods and should not be seen as separate from them. The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. It is important that children acquire secure mental methods of calculation and one efficient written method of calculation for addition, subtraction, multiplication and division which they know they can rely on when mental methods are not appropriate.

119. This document identifies progression in calculation strategies from year 1 to year 6, and encompasses concrete, pictorial and abstract methods. Thus, it adheres to the maths mastery approach. It has been devised to meet the requirements of the National Curriculum 2014 for the teaching and learning of mathematics. By the end of year 6, children should be able to choose the most appropriate approach to solve a problem: making a choice between using jottings (an extended written method), an efficient written method or a mental method.

120. The ideal pen and paper methods in the 4 operations and for working with fractions are efficient, accurate and clear. These can be found in both the National Curriculum appendices and the Star maths progression map. The resulting neatness and logical approach helps to minimise the risk of pupils making accidental errors. Other, informal, methods, some of which may involve physical resources, can be useful for revealing underlying principles and concept, but should only be used for a short time - as a bridge to formal methods - because pupils need to be given adequate opportunities to learn, rehearse and master formal methods, such as column subtraction and long division.

121. If a particular scheme is used to drive the structure, progression and content of sequences of learning in mathematics within a school, then it may be necessary to refine this calculation policy to suite the exact needs and approach of the scheme. In these cases, the most important mastery considerations are:

- To ensure that CPA approaches to the teaching of calculation methods are consistent throughout all year groups;
- To ensure variation – pupils should be exposed to more than one way of representing a calculation.
- To support pupils to move towards formal written methods of calculation

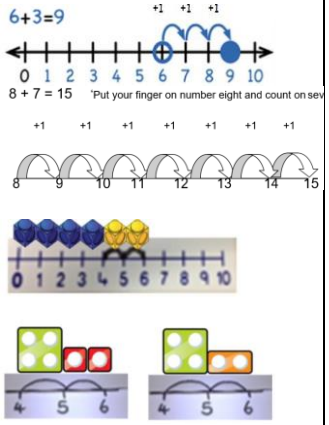
122. The mathematical language used to describe each calculation should be encouraged throughout. The chapter entitled *'Developing Mathematical Vocabulary and Definitions'* provides an explanation of each mathematical term described in the policy.

Addition

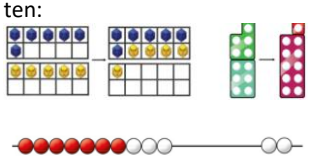
Year 1

Addition of 1- and 2-digit numbers to 20

Use numbered number lines and practical apparatus to add by counting on in ones. Encourage children to start with the larger number and count on:



Use Numicon, bead strings and tens frames to introduce addition by bridging through ten:

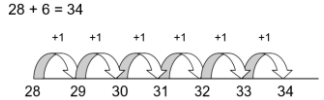


7 add 3 makes 10.
So, 7 add 5 is 10 and 2 more.

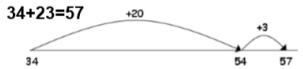
Year 2

Addition of numbers with up to 2 digits

Continue using number lines where required:



Move onto blank number lines -
1. First counting on in 10s and 1s.
2. Become more efficient by adding the 1s in one jump.
3. Followed by adding both the 10s and the 1s in one jump:



Also use blank number lines in conjunction with 100-squares to jump in 10s:

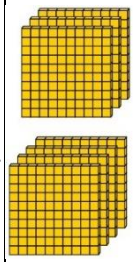
Use known bonds to add 10s:

I know that 4 + 3 = 7. So, I know that 4 tens add 3 tens is 7 tens.

Year 3

Addition of numbers with up to 3 digits

Adding 100s:
Use known facts and unitising to add multiples of 100.

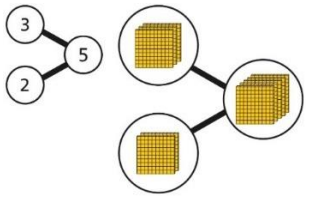


3 + 4 = 7

3 hundreds + 4 hundreds = 7 hundreds

300 + 400 = 700

Use the part-whole model to support abstract understanding of this:

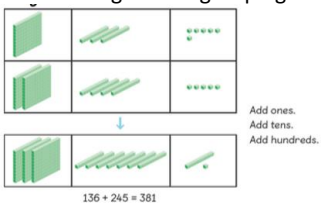


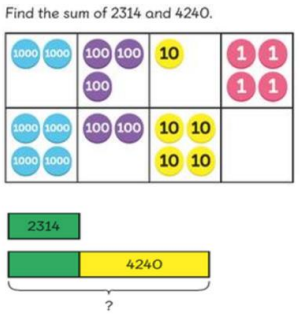
3 + 2 = 5 *300 + 200 = 500*

Year 4

Addition of numbers with up to 4 digits

Reinforce the column method by using concrete methods, including base ten equipment, starting with no regrouping and then moving onto regrouping:

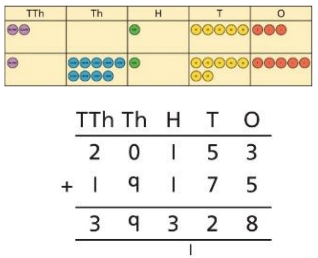




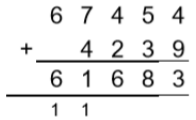
Year 5

Addition of numbers with more than 4 digits

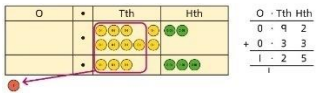
Represent additions using place value equipment on a place value grid alongside written methods.



Use the compact method:
67454+4239=



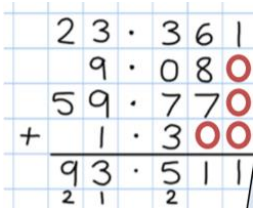
Introduce decimal addition through the use of base ten equipment, representing exchange where necessary.



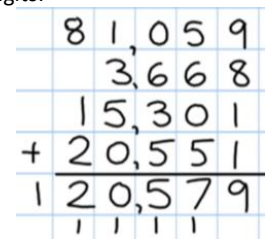
Year 6

Addition of numbers of increasing complexity and decimal value

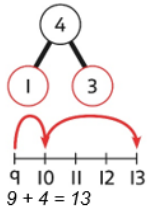
Use the compact method, illustrating the addition of more complex decimals with differing numbers of decimal places:



Practice adding several numbers with more than 4 digits:

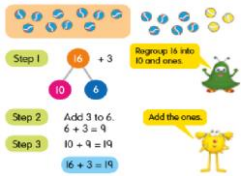


Support bridging through 10 with accompanying part-whole models:

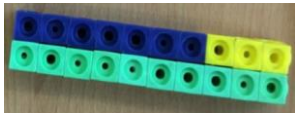


9 + 4 = 13

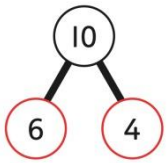
Introduce regrouping into 10s and 1s:



Introduce the bar model:

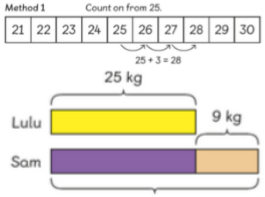


Use the part-whole model to introduce abstract methods:

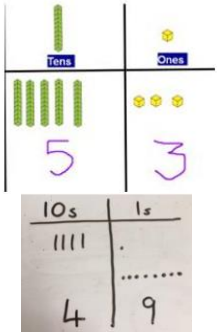


6 + 4 = 10

Use a range of resources and pictorial methods, including bar modelling:



Use base ten apparatus to build understanding of 2-digit places value. Move onto drawing the rods pictorially:

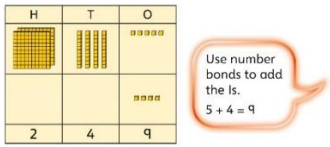


Move onto expanded method as an intermediary to the compact method if required:

$$\begin{array}{rcl} 42 & = & 40 + 2 \\ +\ 36 & & 30 + 6 \\ & & 70 + 8 \end{array} \quad = 78$$

Culminate in compact method, supported by base ten apparatus where required:

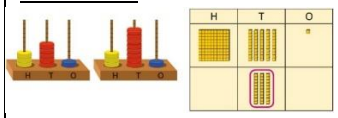
<u>Adding with no exchanging or bridging using concrete objects, base ten apparatus and number lines. Progress from adding 1s to adding 2/3 digit numbers:</u>



245 + 4 5 + 4 = 9

245 + 4 = 249

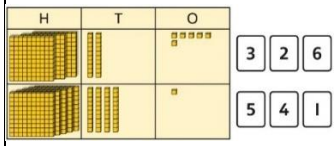
<u>351 + 30 = ?</u>



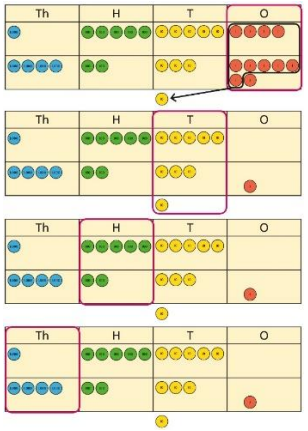
5 tens + 3 tens = 8 tens

351 + 30 = 381

<u>326 + 541 = ?</u>



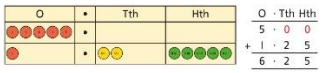
Reinforce exchanging with a range of place value equipment, including where there are multiple exchanges:



Use expanded method as an intermediary to compact method if required, illustrating place value:

$$\begin{array}{rcccl} & 4 & 2 & 5 & 6 \\ +\ & 1 & 9 & 8 & 7 \\ \hline & & & 1 & 3 \quad \text{add ones} \\ & & 1 & 3 & 0 \quad \text{add tens} \\ & 1 & 1 & 0 & 0 \quad \text{add hundreds} \\ +\ & 5 & 0 & 0 & 0 \quad \text{add thousands} \\ \hline & 6 & 2 & 4 & 3 \end{array}$$

Include examples where the numbers of decimal places are different.

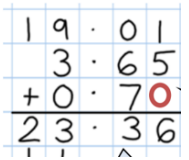


Apply this to numbers with different numbers of decimal places using abstract column method:

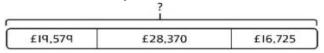
£23.59+£4.38=

$$\begin{array}{r} \pounds\ 2\ 3\ .\ 5\ 9 \\ +\quad 4\ .\ 3\ 8 \\ \hline 2\ 7\ .\ 9\ 7 \\ \hline 1 \end{array}$$

Teach how to align decimal places correctly:

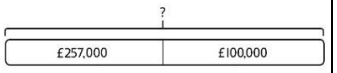


Use bar models to represent and understand problems:



Use a bar model to support thinking in addition problems.

257,000 + 99,000 = ?



I added 100 thousands then subtracted 1 thousand.

257 thousands + 100 thousands = 357 thousands

257,000 + 100,000 = 357,000

357,000 – 1,000 = 356,000

So, 257,000 + 99,000 = 356,000

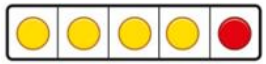
Represent and use number bonds and related subtraction facts within 20

Break apart a group and put back together to find and form number bonds.

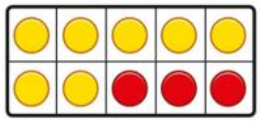
3 + 4 = 7

6 = 2 + 4

Use five and ten frames to represent key number bonds.

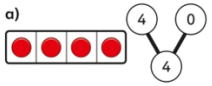


5 = 4 + 1

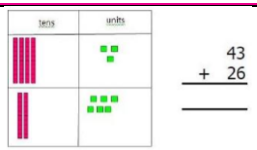


10 = 7 + 3

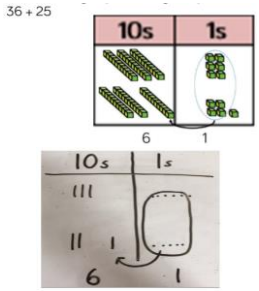
Use a part-whole model alongside other representations to find number bonds. Make sure to include examples where one of the parts is zero.

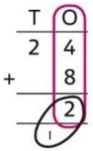


4 + 0 = 4
3 + 1 = 4



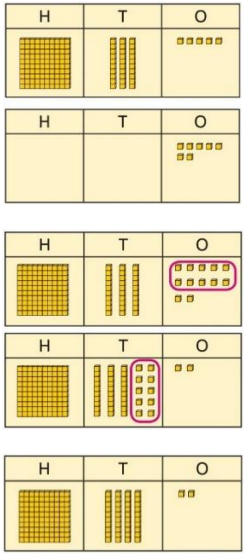
Use both concrete and pictorial methods to <u>for regrouping,</u> focusing on base ten equipment:



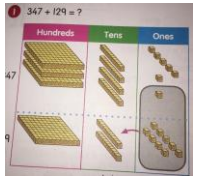


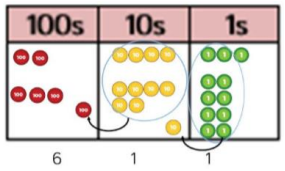
Culminate in the compact abstract method for regrouping, supported by base ten apparatus where required:

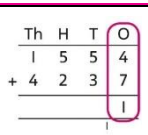
<u>Adding with exchanging, using base ten equipment - concrete and pictorial methods. Begin with adding 1s and then adding 2/3 digit numbers:</u>

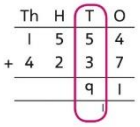


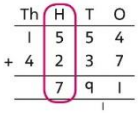
135 + 7 = 142

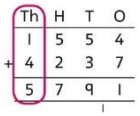






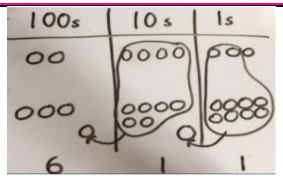
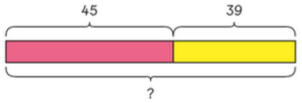
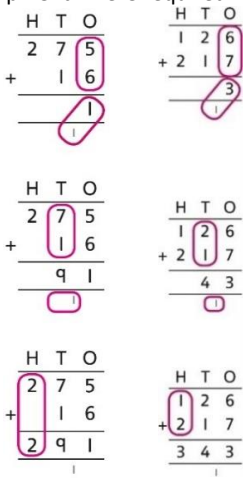




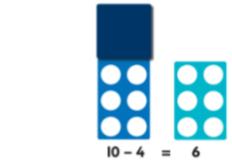
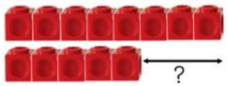
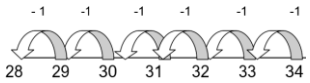
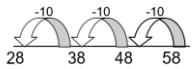
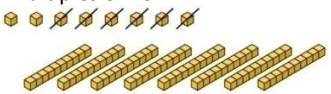
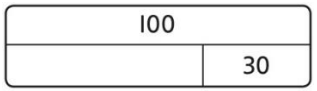
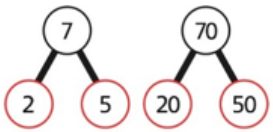
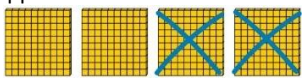
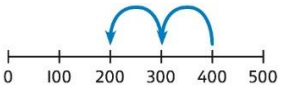
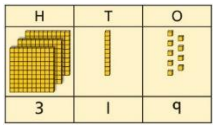
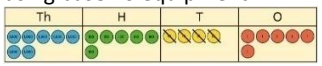
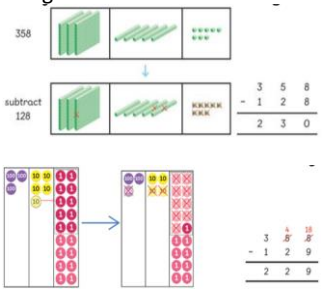
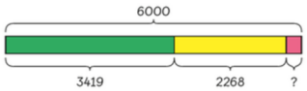
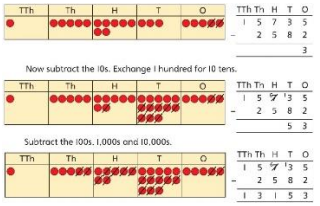
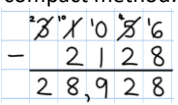
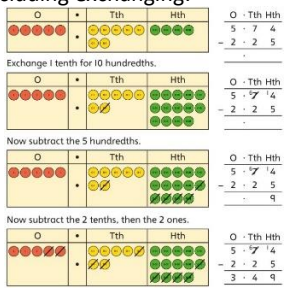
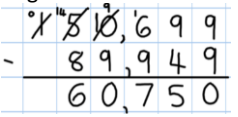
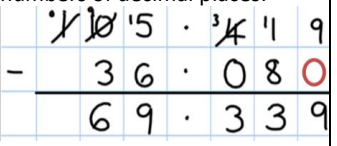


Culminate with compact method, supported by base ten equipment where required. Include examples that exchange in more than one column

Apply this method to money and measures.

		  Use expanded method prior to the compact method as an additional step if required: 2 6 7 + 1 8 5 1 2 1 4 0 3 0 0 4 5 2 Culminate in compact method, supported by base ten equipment where required:  *This should include where multiple exchanging is needed.*		

Subtraction

Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Subtraction of 1- and 2-digit numbers to 20 Consolidate understanding of subtraction using concrete and pictorial methods, showing subtraction on bead strings, Numicon, cubes and number tracks:    9 – 5 = 4 'Put your finger on number nine. Count back five.' Children draw and cross out or use counters to represent objects from a problem.  9 – ☐ = ☐ There are ☐ children left.	**Subtraction of numbers with up to 2 digits** Continue counting back in 1s and 10s on number lines: 34 - 6 = 28  58 - 30 = 28  Use known bonds to subtract multiples of 10:  *8 subtract 6 is 2. So, 8 tens subtract 6 tens is 2 tens.*  *10 – 3 = 7. So, 10 tens subtract 3 tens is 7 tens.*  	**Subtraction of numbers with up to 3 digits** Subtracting 100s: Use known facts a to subtract multiples of 100 using CPA approaches:  *4 – 2 = 2* *400 – 200 = 200*  Use a range of concrete methods, and then pictorial methods to represent subtraction with 3 digits, including base ten equipment, bar modelling and the part-whole model. Start with no regrouping/subtracting 1s and move onto regrouping/subtracting 3-digit numbers:  *319 – 4 = ?*	**Subtraction of numbers with up to 4 digits** Support use of mental methods where appropriate, using base 10 equipment:  *7,646 – 40 = 7,606* Reinforce the column method by using concrete methods:  Use pictorial methods, including bar modelling:  Move onto expanded method as an intermediary to compact method if required, illustrating place value:	**Subtraction of numbers with more than 4 digits** Represent the stages of the calculation using place value equipment on a grid alongside the calculation, including exchanges where required. *15,735 – 2,582 = 13,153*  Use the compact method:  Use a place value grid and pictorial methods to support subtraction of decimals, including exchanging:  	**Subtraction of numbers of increasing complexity and decimal values** Use the compact method with larger numbers:  illustrate the subtraction of more complex decimals with differing numbers of decimal places: 

Progress to a marked number line:

12 - 6 = 6

'Put your finger on number twelve and count back six.'

Use the part-whole model to support abstract methods:

There are 9 bean bags altogether. How many bean bags does Ruby have on her head?

Use the part-whole method to introduce regrouping into 10s and 1s:

Step 2 Subtract 3 from 7.
7 – 3 = 4

Step 3 10 + 4 = 14

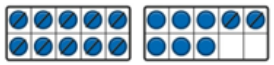
17 – 3 = 14

Peter has 14 toy cars left.

Subtract 10s and 1s with the support of ten frames also.

For example: 18 – 12

Use ten frames to represent the efficient method of subtracting 12.

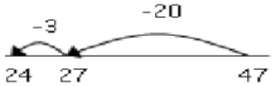


First subtract the 10, then subtract 2.

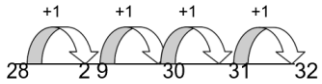
Use complimentary addition to introduce the idea of differences, using small differences only:

Continue using blank number lines with increasingly more efficient jumps -

1. First count back in 10s & 1s.
2. Become more efficient by subtracting the 1s in one jump (by using the known fact 7 – 3 = 4).
3. Subtract the 10s in one jump and the 1s in one jump:

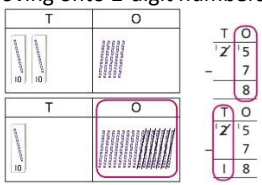


Continue using number lines to find small differences:

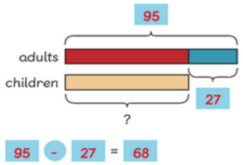


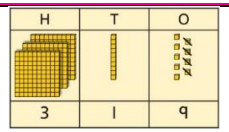
'The difference between 28 and 32 is 4.'

Subtract single-digit numbers using exchanging, before moving onto 2-digit numbers:

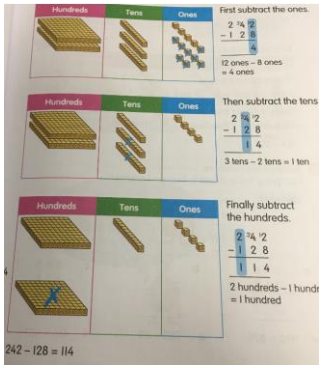


Use a range of concrete and pictorial methods, including bar modelling:

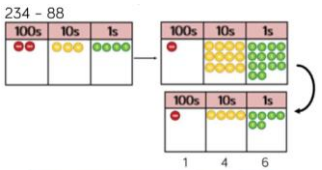


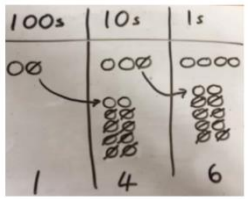


9 – 4 = 5 *319 – 4 = 315*

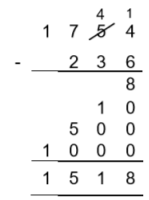


234 - 88

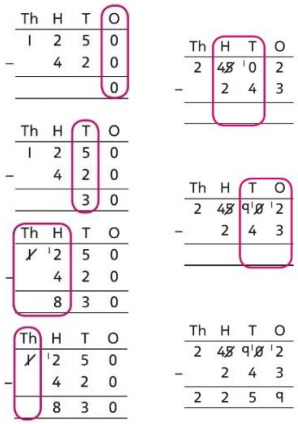




Use bar models to represent subtractions. 'Find the difference' is represented as two bars for comparison.



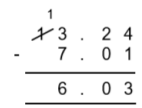
Culminate with compact method, including where there are multiple exchanges:



Apply this method to money and measures.

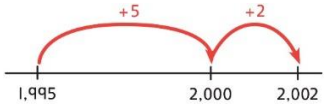
Apply column method to numbers with different numbers of decimal places:

13.24-7.01=

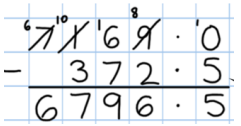


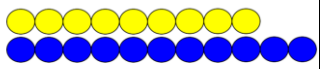
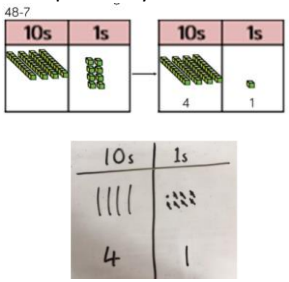
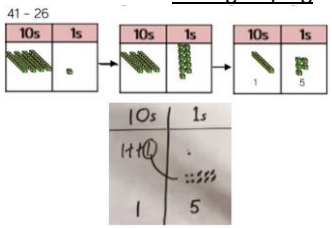
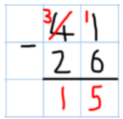
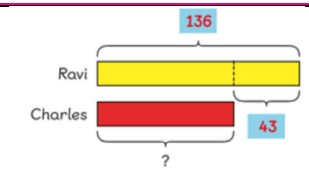
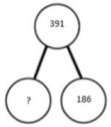
To subtract two large numbers that are close, children find the difference by counting on:

2,002 – 1,995 = ?

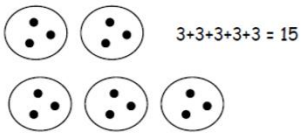
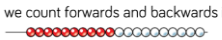
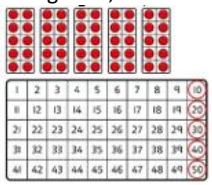
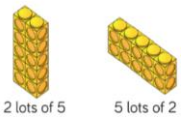
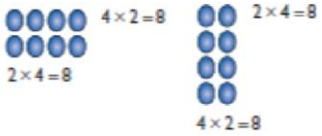
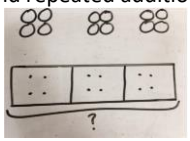
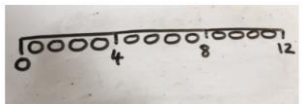
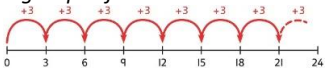
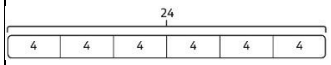
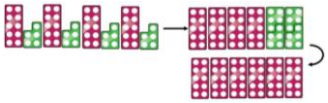
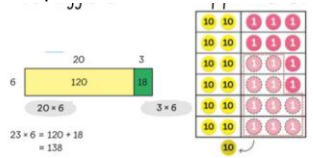
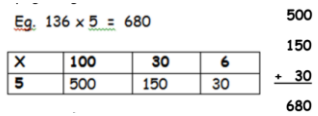
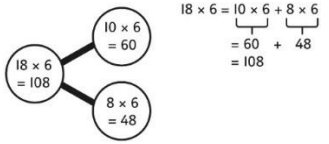
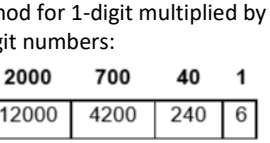
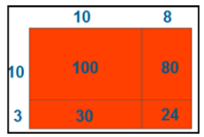
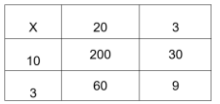
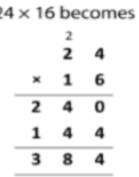
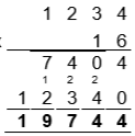
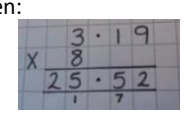


Teach how to align decimal places correctly and to add place value holders, including with decimals:



11 – 9 = 2  The difference between nine and eleven is two.	Use base ten apparatus to build understanding of 2-digit place value. Move onto drawing the rods pictorially:  Move onto expanded method as an intermediary to the compact method with no regrouping if required: 89 = 80 + 9 - 57 = 50 + 7 = 30 + 2 =32 Use concrete methods, and then pictorial methods for regrouping:  Culminate in the abstract compact method for regrouping, supported by base ten equipment where required:  	 136 - 43 = 93  391 186 ? Culminate with abstract compact method, supported by base ten equipment where required: 2 1 234 - 88 6			

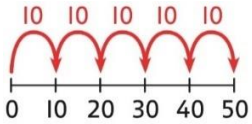
Multiplication

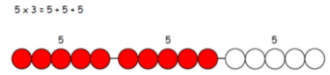
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Solve one-step problems involving multiplication Use a variety of concrete methods and pictorial methods (e.g. bead strings, cubes, real objects), to multiply by grouping (including the use of repeated addition):   'Three pots of ten crayons. How many crayons altogether? 10, 20, 30' There are 3 sweets in one bag. How many sweets are in 5 bags altogether?  Can we count forwards and backwards in tens?  Find the totals of equal groups by counting in 2s, 5s and 10s: 	**Calculate mathematical statements for multiplication and within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs** Use arrays with concrete methods to multiply two 1-digit numbers, including to show commutativity: 2 × 5 = 5 × 2   Employ the use of variety of pictorial and abstract methods, including the bar model, 100-square, bead strings, number lines and repeated addition:  	**Multiply two-digit numbers by one-digit numbers** Cement understanding of link between repeated addition and multiplication. *8 groups of 3 is 24.*  *3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = 24* *8 × 3 = 24* A bar model may represent multiplications as equal groups.  *6 × 4 = 24* Use concrete methods such as Numicon, base ten apparatus or place value counters to multiply by partitioning:  Concrete methods can be replaced with pictorial methods for partitioning, including the part-whole model: *23 x 3 = ?*	Use a variety of CPA methods to develop understanding of multiplication calculations:  Use partitioning in multiplication through the use of the grid method and part-whole models: Eg. 136 x 5 = 680   Use expanded short multiplication as a precursor to short multiplication if required: 127 x 6 = 762 127 x 6 42 (6x7) + 120 (6x20) 600 (6x100) 762	**Multiply numbers up to 4 digits by a 1- or 2-digit number, including long multiplication for 2-digit numbers** Continue developing the grid method for 1-digit multiplied by 4-digit numbers:  Move onto short multiplication: 2741 × 6 becomes  Answer: 16 446 Introduce the grid method for long multiplication to illustrate the relationship between the answers in each individual row:  23 x 13 = (20 + 3) x (10 + 3) = 299  230 + 69 299	**Multiply numbers up to 4 digits by a 2-digit number, including long multiplication for 2-digit numbers** The same as year 5, progressing to long multiplication once pupils are secure with grid method: 24 × 16 becomes  Answer: 384 Introduce long multiplication for 4-digit numbers multiplied by 2-digit numbers:  When applying short multiplication to decimals, ensure the decimal point is in line, with all values carefully written:  Here, briefly return to grid method for multiplying decimals if required.

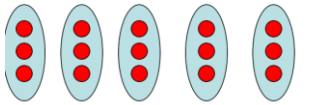
Use arrays to support early multiplication:

'Five groups of two faces. How many faces altogether? 2, 4, 6, 8, 10'
'Two groups of five faces. How many faces altogether? 5, 10'

Use a number line to support repeated addition through counting in 2s, 5s and 10s:

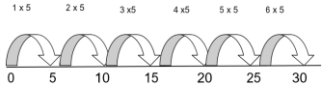




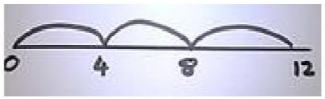


'5 groups of 3' '5 lots of 3' '3 + 3 + 3 + 3 +3 = 15'
'5 times 3' '3 multiplied by 5' '5 x 3 = 15' '3 x 5 = 15'

Use pictorial methods to support the calculation of mathematical statements:



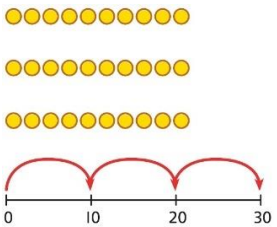
3 × 4 = 12

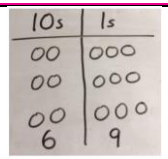


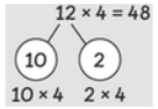
Learning the 2x, 5x & 10x tables

Understand how to relate counting in unitised groups and repeated addition with knowing key times-table facts.

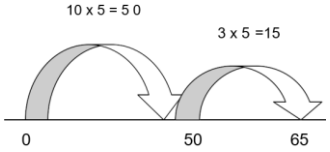
10 + 10 + 10 = 30 *3 × 10 = 30*





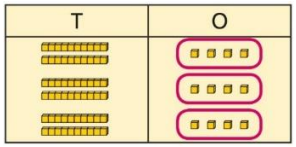


Partitioning can also be demonstrated on a number line:

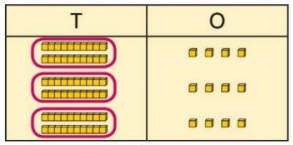


Use place value to support how partitioning is linked with multiplying by a 2-digit number.

3 × 24 = ?



3 × 4 = 12



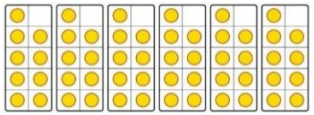
3 × 20 = 60
60 + 12 = 72
3 × 24 = 72

Culminate in short multiplication:

$$\begin{array}{r} 3\ 4\ 2 \\ \times\quad\ 7 \\ \hline 2\ 3\ 9\ 4 \\ \ \ 2\ 1 \end{array}$$

Understanding all times-tables up to 12 × 12

Represent the relationship that the 9, 11 and 12 times tables have with the 10 times table.



2 × 11 = 20 + 2
3 × 11 = 30 + 3
4 × 11 = 40 + 4

Understand how times-tables relate to counting patterns

Understand links between the ×3 table, ×6 table and ×9 table:

5 × 6 is double 5 × 3

×5 table and ×6 table:

I know that 7 × 5 = 35
so I know that 7 × 6 = 35 + 7.

×5 table and ×7 table:

3 × 7 = 3 × 5 + 3 × 2

Use expanded long multiplication if pupils have difficultly transitioning straight to vertical long multiplication:

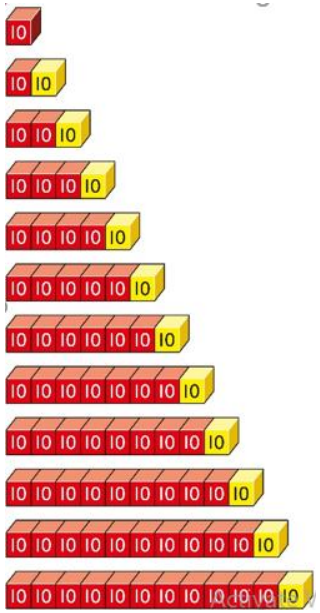
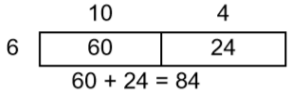
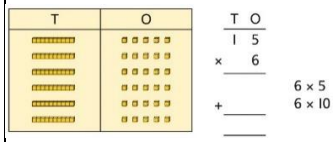
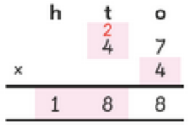
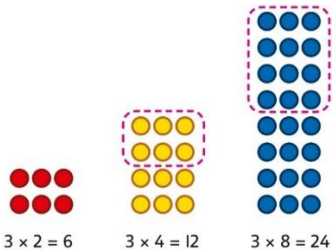
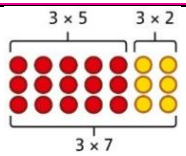
23 x 13 = 299

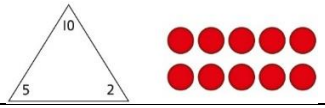
$$\begin{array}{r} 2\ 3 \\ \times 1\ 3 \\ \hline \ \ 9 \quad (3 \times 3) \\ 6\ 0 \quad (3 \times 20) \\ +\ 3\ 0 \quad (10 \times 3) \\ 2\ 0\ 0 \quad (10 \times 20) \\ \hline 2\ 9\ 9 \end{array}$$

Culminate in vertical long multiplication once pupils are secure, including multiplying 3- and 4-digit numbers by 2-digit numbers:

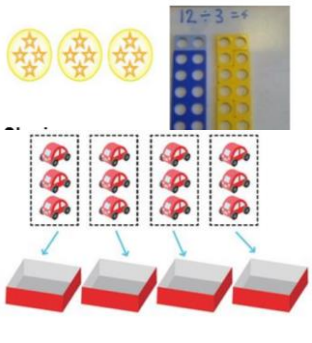
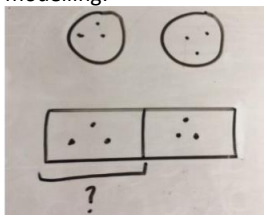
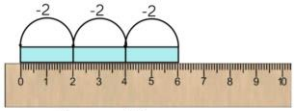
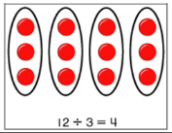
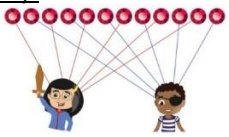
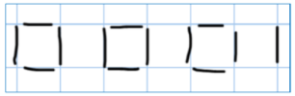
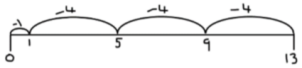
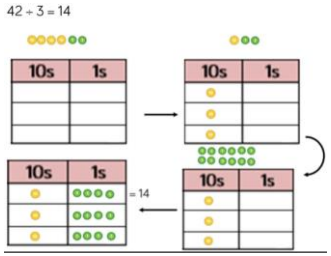
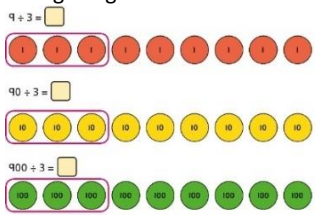
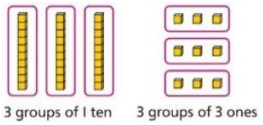
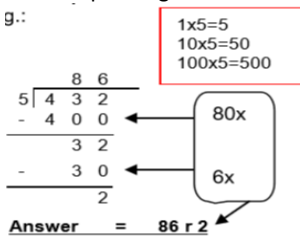
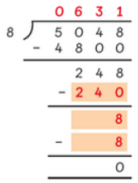
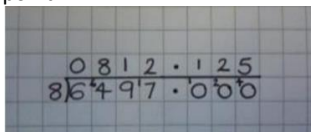
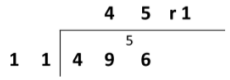
$$\begin{array}{r} 5\ 6 \\ \times\ 2\ 7 \\ \hline 3\ 9^{4}2 \\ +\ 1\ 1^{1}2\ 0 \\ \hline 1\ 5\ 1\ 2 \\ 1 \end{array} \qquad \begin{array}{r} 1\ 2\ 4 \\ \times\quad 2\ 6 \\ \hline 7\ 4\ 4 \\ {}^{1}\ {}^{2} \\ 2\ 4\ 8\ 0 \\ \hline 3\ 2\ 2\ 4 \\ 1\ 1 \end{array}$$

If pupils are making errors, move back to expanded long multiplication or grid method as appropriate.

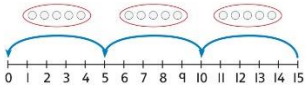
	Understand how the times-tables increase and contain patterns. 	Introduce the abstract grid method, using partitioning:  Expanded column method may be used as a bridge to compact method, but children must understand the link with place value and exchange.  Culminate in short multiplication:  **<u>Understanding and using times tables</u>** Children understand how the ×2, ×4 and ×8 tables are related through repeated doubling.  	 ×9 table and ×10 table: *6 × 10 = 60* *6 × 9 = 60 − 6*		

		Children understand the relationship between related multiplication and division facts in known times-tables. 2 × 5 = 10 5 × 2 = 10 10 ÷ 5 = 2 10 ÷ 2 = 5 			

Division

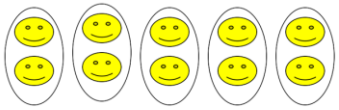
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Solve one-step problems involving division Make equal groups using a variety of concrete methods first, including finding a half and a quarter by grouping into 2 and 4: How many groups of 4 can be made with 12 stars? 3  'Share these eight apples equally between two children. How many apples will each child have?'  Represent grouping and sharing using pictorial methods, including bar modelling: 	**Calculate mathematical statements for division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs** Continue using a range of concrete methods to group objects, including the concrete representation of repeated subtraction, arrays of objects and bead strings:  3 groups of 2  12 ÷ 3 = 4 **15÷5=3** Children should understand this number sentence as '*How many groups of 5 make 15?*' And on a bead string  Develop division as **sharing** equally: 	**Calculate mathematical statements for division, progressing to short division** Introduce 2-digit numbers divided by 1-digit numbers with remainders using the CPA approach, starting with practical resources such as lollipop sticks or Cuisenaire rods:  There are 3 whole squares, with 1 left over.  '3 groups of 4, with 1 left over'  Before introducing the formal method short division, introduce sharing using place value counters: 	**Calculate mathematical statements, including short division for 3-digit numbers divided by 1-digit numbers** Divide multiples of 10 and 100 by a single digit:  *9 ÷ 3 = 3* *9 tens divided by 3 is 3 tens.* *9 hundreds divided by 3 is 3 hundreds.* Support division of 2- and 3- digit numbers through the use of partitioning, using pictorial and abstract methods: *39 ÷ 3 = ?*  3 groups of I ten 3 groups of 3 ones *39 = 30 + 9* *30 ÷ 3 = 10* *9 ÷ 3 = 3* *39 ÷ 3 = 13*	**Divide numbers up to 4 digits by a 1-digit number using the formal written method of short division and interpret remainders appropriately for the context** Continue using chunking to divide 3-digit numbers by 1-digit numbers, beginning to subtract larger amounts of the divisor each time. Write a useful facts box on the side to support this. Remainders need to be shown as a whole number, or rounded up or down depending on context:  1x5=5 10x5=50 100x5=500 80x 6x Answer = 86 r 2 Move onto chunking with 4-digit numbers: 	**Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context** Continue using the methods taught in year 5, culminating in the consistent use of short division, including the representation of the remainder as a decimal, according to context. Here, the number of decimal places required is added to the divisor, and the 'carrying over' continues beyond the decimal point:  Also use short division, interpreting the remainder as a fraction: 496 ÷ 11 becomes  Answer: $45\frac{1}{11}$

Children may relate this to counting back in steps of 2, 5 or 10:



Include the use of arrays to support early division:

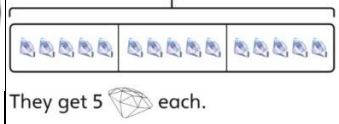
'How many faces altogether? How many groups of two?'



'Five groups of two'

12 shared equally between 2.
They get 6 each.

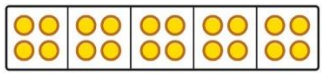
Start to understand how this also relates to grouping. To share equally between 3 people, take a group of 3 and give 1 to each person. Keep going until all the objects have been shared.



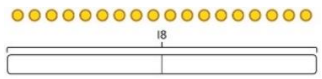
They get 5 each.

Represent the objects **shared** into equal parts using a bar model

20 shared into 5 equal parts.
There are 4 in each part.

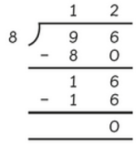


Use a bar model to support understanding of division as **sharing.**

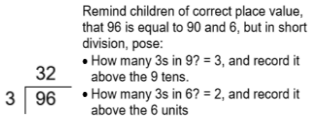


18 ÷ 2 = 9

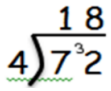
Show the written method of 'chunking', using division facts to take chunks away, relating this to short division:



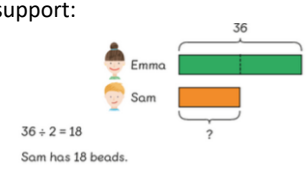
Move children onto the formal written method of short division once full understanding of previous methods is secure. Initially use questions with no calculation of remainders in any stage of the calculation:



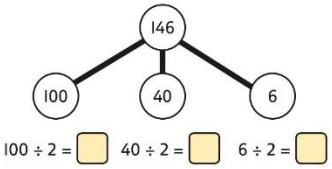
Move onto questions containing remainders within the calculation <u>but not the answer</u> – pupils need to 'carry over':



When solving problems, visual representations such as bar modelling can still be used to support:

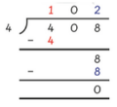


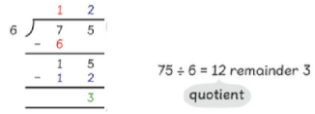
142 ÷ 2 = ?



100 ÷ 2 = 50
40 ÷ 2 = 20
6 ÷ 2 = 3
50 + 20 + 3 = 73
142 ÷ 2 = 73

Continue to build understanding using chunking - the abstract written method which is the precursor to short division:

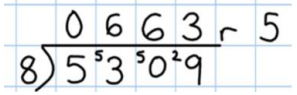




Move onto the same chunking method with 3-digit numbers, moving onto remainders if appropriate:

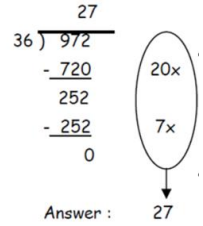
Culminate in short division of 3-digit numbers by 1-digit numbers, with no remainders, but moving beyond this with remainders if appropriate:

Progress to short division of 4-digit numbers by 1-digit numbers, with remainders. Note that division needs to have a real life problem solving context, where pupils consider the meaning of the remainder and how to express it - i.e. as a fraction, a decimal, or as a rounded number or value, depending upon the context of the problem:

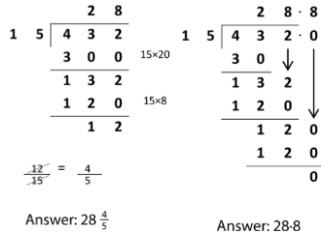


<u>Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context</u>

Introduce dividing up to 4 digits by a two-digit whole number using chunking in the first instance, continuing to use a useful facts box. Start with simple chunks (x10, 100 etc), and then move onto more efficient chunks:



Interpret long division answers as fractions and as decimals, according to context:

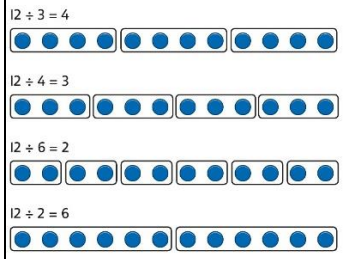


Develop concept of division as grouping:

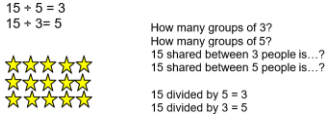
Understand how to make equal groups from a whole:

8 divided into 4 equal groups.
There are 2 in each group

Understand the relationship between grouping and the division statements:

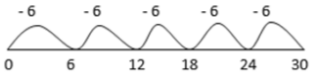


Continue using arrays to support early division:



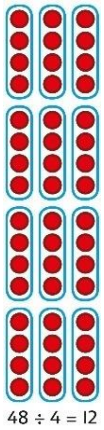
Repeated subtraction can also be represented using abstract number lines:

30 ÷6



Using times tables to divide

Use knowledge of known times-tables to calculate divisions:

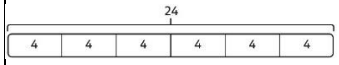


48 divided into groups of 4.
There are 12 groups.

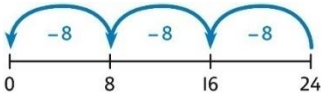
4 × 12 = 48
48 ÷ 4 = 12

48 ÷ 4 = 12

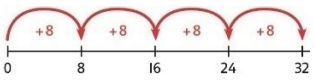
Use bar models and number lines to support application of times tables to division:



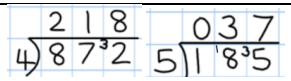
24 ÷ 4 = 6 *24 ÷ 6 = 4*



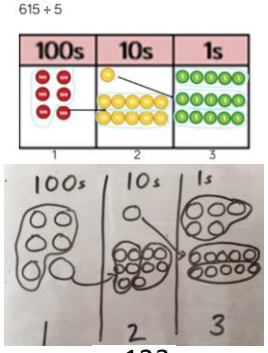
24 ÷ 8 = 3



32 ÷ 8 = 4



Continue using a CPA approach for dividing 3-digit numbers by 1-digit numbers, to support understanding of division with some pupils where required:



Understand the relationship between division and multiplication using times tables

Use objects to explore families of multiplication and division facts:

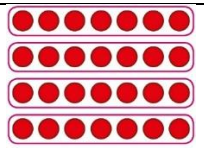
4 × 6 = 24

24 is 6 groups of 4.
24 is 4 groups of 6.

24 divided by 6 is 4.

24 divided by 4 is 6.

Represent divisions using an array:

	Record as mathematical statements once secure: 15 ÷ 5 = 3 Associate the law of commutativity to show link between multiplication and division: 5 x 2 = 10 ——— 10 ÷ 2 = 5 2 x 5 = 10 ——— 10 ÷ 5 = 2		 28 ÷ 7 = 4 Understand families of related multiplication and division facts: *I know that 5 × 7 = 35 so I know all these facts:* *5 × 7 = 35* *7 × 5 = 35* *35 = 5 × 7* *35 = 7 × 5* *35 ÷ 5 = 7* *35 ÷ 7 = 5* *7 = 35 ÷ 5* 5 = 35 ÷ 7		

Trust Long Term Planning

123. The exact lengths and order of maths units within each term is often derived by school-level decisions, influenced by the precise needs of groups and cohorts of pupils. This is particularly prevalent in the spring and summer terms of years 2 and 6, when exam preparation increasingly takes precedence.

124. However, where possible, the following termly Trust 'checkpoints' are adhered to. These checkpoints denote the areas of mathematics that should be covered within a term. These align seamlessly with the long-term overviews from both White Rose Maths and Power Maths, which are themselves almost identical to each other, and ensure maximum consistency across the Trust for termly inter-school moderation activities:

YEAR 1		
Autumn	**Spring**	**Summer**
Place Value (within 10)	Addition and Subtraction (adding & crossing 10)	Multiplication
Addition and Subtraction	Place Value (within 50)	Division
Properties of Shape	Length and Height	Fractions
Place Value (within 20)	Weight and Volume	Position and Direction
		Numbers to 100
		Time
		Money
YEAR 2		
Autumn	**Spring**	**Summer**
Place Value (to 100)	Division	Position and Direction
Addition and Subtraction	Statistics	Problem Solving & Efficient Methods
Money	Length and Height	Time
Multiplication	Properties of Shape	Mass, Capacity & Temperature
	Fractions	
YEAR 3		
Autumn	**Spring**	**Summer**
Place Value (within 1000)	Multiplication & Division (written)	Fractions (equivalence, ordering, addition and subtraction)
Addition and Subtraction (written)	Measurement: Money	Time
Multiplication & Division (mental)	Statistics	Properties of Shape
	Length & Perimeter	Mass and Capacity
	Fractions (tenths, fractions of objects)	
YEAR 4		
Autumn	**Spring**	**Summer**
Place value – 4-digit numbers	Multiplication and Division - written	Decimals – writing and comparing
Addition and subtraction	Area	Money
Perimeter	Fractions	Time
Multiplication and Division – mental, divide by 10, 100	Decimals - tenths, hundredths, division by 10 and 100	Statistics
		Angles and 2D Shapes
		Position and Direction

YEAR 5		
Autumn	**Spring**	**Summer**
Place Value to 1,000,000	Multiplication and Division - written)	Decimals - adding, subtracting, multiply and divide by 10, 100, 1000
Addition and Subtraction - written	Fractions	Angles
Statistics	Decimals and Percentages	Properties of Shape
Multiplication and Division - mental, times and divide by 10, 100, 1000		Position and Direction
Area and Perimeter		Converting Units
		Volume and Capacity
YEAR 6		
Autumn	**Spring**	**Summer**
Place value - within 10,000,000	Decimal	Properties of Shapes
Four Operations	Percentages	Problem Solving
Fractions	Algebra	Statistics
Position and Direction	Converting Units	
	Perimeter, Area and Volume	
	Ratio and Proportion	

Supporting Pupils to Achieve Greater Depth

125. Vital for mastery of the curriculum is the development of deep rather than superficial conceptual understanding. A mastery approach focuses on fewer things in greater depth, rather than relentless, over-rapid progression. It is inevitable that some pupils will grasp concepts more rapidly than others and will need to be stimulated and challenged to ensure continued progression.

126. However, research indicates that these pupils benefit more from enrichment and deepening of content, rather than acceleration into new content. Acceleration is likely to promote superficial understanding, rather than the true depth and rigour of knowledge that is a foundation for higher mathematics.

127. All pupils require depth in their maths learning, but some pupils will go deeper still in their learning and understanding.

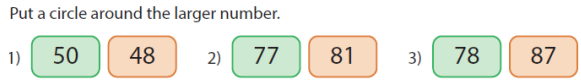
128. Mastery of the curriculum requires that all pupils:

- Use mathematical concepts, facts and procedures appropriately, flexibly and fluently;
- Recall key number facts with speed and accuracy and use them to calculate and work out unknown facts;
- Have sufficient depth of knowledge and understanding to reason and explain mathematical concepts and procedures and use them to solve a variety of problems.

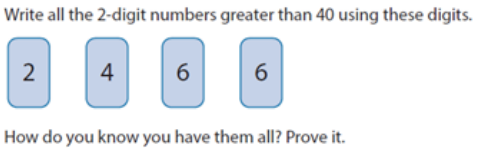
129. Here, a pupil can be said to have fully mastered a mathematical concept when he or she can:

- Describe it in his or her own words;
- Represent it in a variety of ways (e.g. using concrete materials, pictures and symbols);
- Explain it to someone else;
- Make up his or her own examples (and non-examples) of it;
- See connections between it and other facts or ideas;
- Recognise it in new situations and contexts;
- Make use of it in various ways, including in new situations.

130. For pupils whose understanding goes even deeper, developing 'mastery with greater depth' is characterised by pupils' ability to:
    - Solve problems of greater complexity (i.e. where the approach is not immediately obvious), demonstrating creativity and imagination;
    - Independently explore and investigate mathematical contexts and structures, communicate results clearly and systematically explain and generalise the mathematics.
131. Thus, in order to support pupils to achieve mastery with greater depth without accelerating some pupils prematurely into new content, these rapid graspers need to be supported to go deeper within the same content, according to the two bullet points above. They need to be provided with richer, more complex and challenging questions, that contain greater complexity, encourage more independent exploration, require more systematic explanations and necessitate generalisations.
132. The development of these greater depth skills should be modelled, and all this needs to occur within the bounds of the central mastery tenet, that all pupils learn at the same pace and do not accelerate prematurely into new content.
133. A teacher's skill in transforming a mastery question into a mastery with greater depth question will therefore hinge on the ability to introduce one or more of the following additional elements of challenge:
    - Questions with many possible answers – open-ended and requiring independent exploration;
    - Requirement to explain and/or generalize about underlying concepts;
    - Introduction of added complexity - additional steps to problems;
    - Application involving other concepts and/or other areas of maths.
134. The following pairs of mastery/greater depth questions exemplify this transformation, and are taken from the NCETM 'Teaching for Mastery' documents. They cite which element/s of additional greater depth challenge have been employed in order to transform the question into a greater depth version. Through understanding the processes that are occurring below when transforming mastery questions into greater depth ones, teachers can then develop their own abilities to generate greater depth questions, rather than relying on pre-prepared banks of GDS questions.
135. Year 2 Place Value



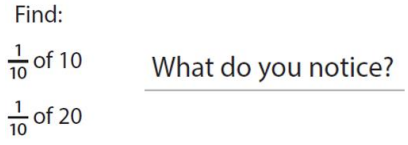
Mastery



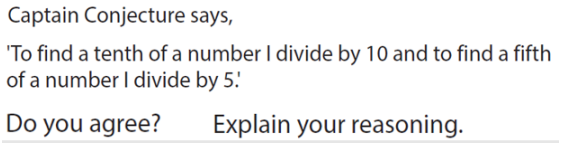
Mastery with Greater Depth

Here, the question has become more independently investigative, with many possible answers, and there is the additional requirement to be more systematic in order to ensure that all possible answers have been found.

136. Year 4 Fractions



Mastery



Mastery with Greater Depth

Here, the greater depth version of this year 4 fraction of amounts focus requires the pupil to explain and generalise about the underlying concepts involved.

137. Year 5 Measurement

Complete this:

$\frac{1}{2}$ kg = ____ g

$\frac{1}{4}$ kg = ____ g

Mastery

True or false?

1·5 kg + 600 g = 2·1 kg + 300 g

32 cm + 1·05 m = 150 cm – 0·13 m

$\frac{3}{4}$ ℓ + 0.05 ℓ = half of 1·6 ℓ

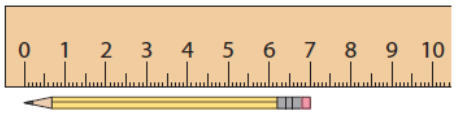
Explain your reasoning.

Mastery with Greater Depth

Here, the greater depth version of this year 5 conversion question has added complexity because it is also introducing the requirement to add measurements as well as convert them. In addition, it also requires mastery of additional areas of maths – decimals.

138. Year 2 Measurement

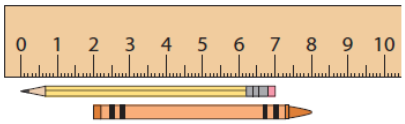
How long is the pencil?



The pencil is _____ cm long.

Mastery

How long is the crayon?



The crayon is _____ cm long.

How much longer is the crayon than the pencil?

Mastery with Greater Depth

Here, the greater depth version of this year 2 conversion question has added complexity because it has incorporated other areas of maths – finding the difference.

139. Year 6 Addition and Subtraction

Choose digits to go in the empty boxes to make these number sentences true.

14 781 – 6 ☐ 53 = 8528

23·12 + 22· ☐ = 45·23

Mastery

The greater depth version of this mastery question requires independent exploration and there are many possible answers.

Can you use five of the digits 1 to 9 to make this number sentence true?

☐☐·☐ + ☐·☐ = 31·7

Can you find other sets of five of the digits 1 to 9 that make the sentence true?

Mastery with Greater Depth

Supporting Pupils with Questioning

140. Good questioning techniques are a fundamental tool of effective teaching. Questions that guide, support and stimulate pupils in their fluency activities, problem solving, reasoning and investigating are essential in developing confident and able mathematicians.

141. Bloom's Taxonomy outlines six types of thinking, all of which can be related to the types of questions teachers can ask in the classroom. These range from lower order questions (remembering, understanding and applying) to higher order questions (analysing, evaluating and creating). A combination of all six question types should be used in mathematics lessons to both support pupils in developing their mastery of concepts and to allow the teacher to engage in essential assessment for learning activities.

142. The following are exemplar questions to support pupils in remembering, understanding, applying, analysing, evaluating and creating. All questions below are given within the context of particular maths foci but can easily be transferred to other areas of maths.

<u>Remembering</u>

What is a factor?

What is a prime number?

What is an even number?

What are the first 5 square numbers?

What comes first in BIDMAS?

What is a denominator? What is a numerator?

What percentage is equivalent to a quarter?

If ones are just before the decimal point, what is just after?

Can you give all the facts you know about 10?

Describe the properties of a triangle.

Identify the similarities/differences between a square and a rectangle.

What does a number have to end in to be even?

What does mean average mean?

What does the 5 represent in 53?

What comes next – 10, 20, 30, 40...?

How many 2s are in 16?

List as many multiples of 5 as possible.

Which of the following times tables questions are false?

<u>Understanding</u>

Why is 7 prime?

Why is -4 larger than -7?

Why is a square a rectangle?

Why are 1, 4, 9, 16 called square numbers?

How is BIDMAS used?

How would you explain to someone how to multiply a decimal by 10 … , how to divide a decimal by 100?

Why are equivalent fractions important when adding or subtracting fractions?

Explain how to simplify 2/6.

What denominators make it easy to change a fraction to a decimal?

What is the purpose of the decimal point?

Can you compare a rhombus with a kite?

Can you give a reason for needing to exchange a ten for ones?

Explain how to find the difference between the maximum and minimum values in this bar chart.

Applying

What are the prime factors of 125? 81? 343?

What do you notice?

Can a prime number be a multiple of 4?

What do you have to add to 0.03 to make 0.7?

Which is the square number closest to 30?

The answer to 5 + 2 x 3 is 21. True or False? Why?

What is a common denominator for adding 1/3 and 1/5?

Give me some examples of numbers that are easy to find one fifth of. What about two fifths?

Calculate 20% of £260.

Talk me through how you would use rounding to estimate the answer to this calculation.

How would you sort these numbers?

How would you divide a number by 10 and then by 100?

How would you represent 9 on this pictogram?

How do you know if a number is in the 10 times table?

Analysing

How do you go about finding the prime factors of a given number?

Which is the square number closest to 30?

Why do we need BIDMAS?

What strategies do you use to find a common denominator when adding or subtracting fractions?

Is there anything special about numbers that are easy to find one fifth of?

Think of as many ways as possible to partition 57.

To calculate 10% of a quantity, you divide it by 10, so to find 20%, you must divide by 20. What is wrong with this statement?

Explain how you can use equivalent sets of %, decimals and fractions that you know, to find other equivalent sets.

How would you go about finding the decimal equivalents of any fraction?

Why do we need half pictures in pictograms?

Explain why 0.3 x 100 is the same as 3 x 10?

I divide a number by 10, and then divide the result by 10. The answer is 0.3. What number did I start with? How do you know?

Can you think of 3 different ways to make a total of 13p using coins?

Evaluating

The sum of two square numbers is 20 – what are they?

What clues do you look for when you are reading a calculation and deciding the order of operations?

How would you find five eighths of a number or quantity?

If I divide one fraction by another, I will always get a smaller answer – true/false? Why? Explain.

Which is bigger 12% of 88 or 88% of 12?

Why is 0 so useful when it is worth nothing?

How would you explain to someone that 0.35 is greater than 0.035?

Multiplying makes numbers bigger. When is this statement true and when is it false?

Why do 2.5 ÷ 10 and 25 ÷ 100 give the same answer?

Creating

Create some questions where you would get the answer wrong if you didn't use BIDMAS.

The answer to a fraction addition question is 2/5. What could be the question?

What percentages can you easily work out in your head? Talk me through a couple of examples.

Give me a percentage question with the answer 20.

Create three conversion questions at different levels of difficulty.

The answer to a rounding question is 20. What could the original, unrounded numbers have been?

Create three decimal questions which all give the same answer.

Create your own number sequence and explain it to your partner.

Create two different Venn diagrams to sort this set of numbers.

Assessment

143. Short-term tests and quizzes taken at the ends of units indicate if a pupil knows more and remembers more - these can be called *end-of-unit composite tests*. Termly tests (i.e NTS) give greater assurance that the learning of content is long term. A mixture of approaches is therefore best: regular tests of recently taught content *and* long term summative assessment are both important in mathematics.
144. The White Rose Maths end-of-block tests are one ideal source of appropriate low-stakes, short-term, end-of-unit testing. For longer term summative testing, Star schools use the Rising Stars NTS termly assessments. These test, cumulatively, all learning to date, as per the Star mathematics long term plans and both the White Rose and Power Maths long term sequences of learning.

Appendix 1a: KS2 Test Framework – Maths Strand Proportions

This table shows the proportions of the total marks in the last four KS2 maths SATS that are attributed to the different maths strands.

	2016	2017	2018	2019
Content Domain	Percentage	Percentage	Percentage	Percentage
Number and Place Value	17	9	10	9
Calculations	17	22	29	34
FDP	15	14	14	20
Ratio and Proportion	4	9	6	7
Algebra	8	9	9	5
Measurement	15	14	13	10
Shapes	12	9	10	6
Position and Direction	4	3	4	4
Statistics	8	11	6	4

Appendix 1b: KS2 Test Framework – Maths Strand Proportions without Paper 1: Arithmetic

This table shows the proportions of the total marks in the last four KS2 maths SATS that are attributed to the different maths strands, for paper 2 and paper 3 only. This table therefore gives a more accurate picture of where the focus is for maths reasoning and problem solving in the KS2 maths SATS, without the disproportionate weighting caused by the dominant calculations strand in paper 1: arithmetic.

	2016	2017	2018	2019
Content Domain	Percentage	Percentage	Percentage	Percentage
Number and Place Value	14	12	9	13
Calculations	23	26	30	26
FDP	17	14	14	13
Ratio and Proportion	6	10	7	6
Algebra	6	7	7	8
Measurement	16	10	16	16
Shapes	6	9	11	9
Position and Direction	3	3	1	6
Statistics	6	10	4	6